Assignment 1

is on website

Due Thurs Jan 22, 8:30am

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2.2 Separable Variables

Ex: Solve $\frac{dy}{dx} = y^2 - 9$

$$\frac{dy}{y^2-9} = dx$$

$$\int \frac{dy}{y^2-9} = \int dx$$

Partial Fractions for $\frac{1}{y^2-9}$

$$\frac{1}{(y-3)(y+3)} = \frac{A}{y-3} + \frac{B}{y+3}$$

$$1 = A(y+3) + B(y-3)$$

Sub $y=-3$: $1 = B(-6) \Rightarrow B = -\frac{1}{6}$

Sub $y=3$: $1 = A(6) \Rightarrow A = \frac{1}{6}$

$$\int \left(\frac{1}{6}\frac{1}{y-3} - \frac{1}{6}\frac{1}{y+3} \right) dy = \int dx$$

$$\frac{1}{6}\ln|y-3| - \frac{1}{6}\ln|y+3| = x + C_1$$

$$\ln|y-3| - \ln|y+3| = 6x + C_2$$

$$\ln\left|\frac{y-3}{y+3}\right| = 6x + C_2$$

$e^{LS} = e^{RS}$:

$$\left|\frac{y-3}{y+3}\right| = e^{6x+C_2}$$

$$\left|\frac{y-3}{y+3}\right| = e^{C_2} e^{6x}$$

$$\frac{y-3}{y+3} = \pm e^{C_2} e^{6x}$$

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$$y-3 = Ce^{6x}(y+3)$$

$$y-3 = Ce^{6x}y + 3Ce^{6x}$$

$$y - Ce^{6x}y = 3 + 3Ce^{6x}$$

$$y(1 - Ce^{6x}) = 3(1 + Ce^{6x})$$

$$y = \frac{3(1+Ce^{6x})}{1-Ce^{6x}}$$

IMPLICIT SOLUTION $\quad y-3 = Ce^{6x}(y+3)$

EXPLICIT SOLUTION (solved for y) $\quad y = \frac{3(1+Ce^{6x})}{1-Ce^{6x}}$

Standard form for implicit solutions:
Get rid of ln, absolute values, fractions

Ex: Find an implicit solution

$$\frac{dy}{dx} = \frac{(x-1)(y+3)}{(x+4)(y-2)}$$

$$\frac{y-2}{y+3}\,dy = \frac{x-1}{x+4}\,dx$$

$$\int \frac{y-2}{y+3}\,dy = \int \frac{x-1}{x+4}\,dx$$

Long Division

$$\begin{array}{r} 1 \\ (y+3)\overline{)\,y-2} \\ -(y+3) \\ \hline -5 \end{array}$$

$$\Rightarrow \frac{y-2}{y+3} = 1 - \frac{5}{y+3}$$

OR $$\frac{y-2}{y+3} = \frac{y+3-5}{y+3} = 1 - \frac{5}{y+3}$$

$$\begin{array}{r} 1 \\ (x+4)\overline{)\,x-1} \\ -(x+4) \\ \hline -5 \end{array}$$

$$\Rightarrow \frac{x-1}{x+4} = 1 - \frac{5}{x+4}$$

OR $$\frac{x-1}{x+4} = \frac{x+4-5}{x+4} = 1 - \frac{5}{x+4}$$

$$\int \left(1 - \frac{5}{y+3}\right) dy = \int \left(1 - \frac{5}{x+4}\right) dx$$

$$y - 5\ln|y+3| = x - 5\ln|x+4| + C_1$$

$$y + \ln|y+3|^{-5} = x + \ln|x+4|^{-5} + C_1$$

$e^{LS} = e^{RS}$:

$$e^{y} \cdot e^{\ln|y+3|^{-5}} = e^{x} \cdot e^{\ln|x+4|^{-5}} \cdot e^{C_1}$$

$$e^{y}|y+3|^{-5} = e^{x}|x+4|^{-5} \cdot e^{C_1}$$

$$e^{y}(y+3)^{-5} = \pm e^{C_1} e^{x}(x+4)^{-5}$$ ✓

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or $$e^{y}(x+4)^{5} = Ce^{x}(y+3)^{5}$$ ✓

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Ex: Find an implicit solution.

$$\begin{cases} (e^{3y} - ye^{y})\frac{dy}{dx} = e^{2y}\cos 5x \\ y(0) = 7 \end{cases}$$

Divide by e^{2y}: $e^{-2y}(e^{3y} - ye^{y})\frac{dy}{dx} = e^{-2y}e^{2y}\cos 5x$

$$(e^{y} - ye^{-y})\,dy = \cos 5x\,dx$$

$$\int (e^{y} - ye^{-y})\,dy = \int \cos 5x\,dx$$

Integration by Parts for $\int -ye^{-y}\,dy$

	D	I
+	$-y$	e^{-y}
−	-1	$-e^{-y}$
	0	e^{-y}

$$\int -ye^{-y}\,dy = ye^{-y} + e^{-y} + C$$

$$e^{y} + ye^{-y} + e^{-y} = \frac{\sin 5x}{5} + C_1$$

Sub $x = 0$, $y = 7$: $e^{7} + 7e^{-7} + e^{-7} = 0 + C_1$

$$e^{7} + 8e^{-7} = C_1$$

$$e^{y} + (y+1)e^{-y} = \frac{\sin 5x}{5} + e^{7} + 8e^{-7}$$

or $$5e^{y} + 5(y+1)e^{-y} = \sin 5x + 5(e^{7} + 8e^{-7})$$