List of HW Problems and Full Solutions: Website
Problems are on D2L
or www.leahhoward.com/252CP.pdf

Do Sugg HW for Sections 1.1 and 1.2
Section 1.2 Omit #25, 27

1.2 IVP Cont'd

Ex: $\dfrac{d^2x}{dt^2} + 9x = 0$ has solution

$$x = C_1 \cos 3t + C_2 \sin 3t.$$

Solve if $x\left(\frac{\pi}{3}\right) = 6$ and $x'\left(\frac{\pi}{3}\right) = 1$.

$$\boxed{x = C_1 \cos 3t + C_2 \sin 3t}$$

$t = \frac{\pi}{3}$, $x = 6$: $6 = C_1 \cos \pi + C_2 \sin \pi$

$$6 = -C_1$$

$$C_1 = -6$$

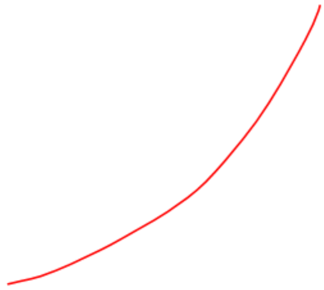
$$\boxed{x = -6 \cos 3t + C_2 \sin 3t}$$

$$x' = 18 \sin 3t + 3C_2 \cos 3t$$

$t = \frac{\pi}{3}$, $x' = 1$: $1 = 18 \sin \pi + 3C_2 \cos \pi$

$$1 = -3C_2$$

$$C_2 = -\frac{1}{3}$$



$$\boxed{x = -6\cos 3t - \frac{1}{3}\sin 3t}$$

<u>Ex</u>: $y = C_1 e^{x} + C_2 e^{-x}$

solves $y'' - y = 0$.

Solve $\begin{cases} y'' - y = 0 \\ y(0) = 2, \quad y'(0) = 3 \end{cases}$

$$\boxed{y = C_1 e^{x} + C_2 e^{-x}}$$

$x = 0$, $y = 2$: $\quad 2 = C_1 + C_2 \qquad (1)$

$$y' = C_1 e^{x} - C_2 e^{-x}$$

$x = 0$, $y' = 3$: $\quad 3 = C_1 - C_2 \qquad (2)$

$$\begin{cases} C_1 + C_2 = 2 \\ C_1 - C_2 = 3 \end{cases}$$

Use any method.

Cramer's Rule

$$C_1 = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-5}{-2} = \frac{5}{2}$$

$$C_2 = \frac{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{1}{-2} = -\frac{1}{2}$$

$$\boxed{y = \frac{5}{2} e^{x} - \frac{1}{2} e^{-x}}$$

2.2 Separable Variables

Ex: Solve $\frac{dy}{dx} = 2xy^2$

"Variables are separated"
"DE is separable"

$$\frac{dy}{y^2} = 2x\,dx$$

$$\int y^{-2}\,dy = \int 2x\,dx$$

$$-y^{-1} = x^2 + C_1$$

$-y^{-1} + C_1 = x^2 + C_2$
would be unnecessary.

$-y^{-1} = x^2 + \cancel{C_2 - C_1}\ C_3$

$$-\frac{1}{y} = x^2 + C_1$$

$$-\frac{1}{x^2+C_1} = y$$

$$y = -\frac{1}{x^2+C}$$

Ex: Solve $\frac{dy}{dx} = \frac{y}{x}$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + C_1$$

$e^{LS} = e^{RS}$:

$$e^{\ln|y|} = e^{\ln|x| + C_1}$$

$$e^{\ln|y|} = e^{\ln|x|} \cdot e^{C_1}$$

$$|y| = |x| e^{C_1}$$

$$y = \pm e^{C_1} x$$

$$y = Cx$$

Ex: Solve $\frac{dy}{dx} = \frac{y}{2-x}$

if $y(0)=6$.

Interval of solution?

$$\frac{dy}{y} = \frac{dx}{2-x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{2-x}$$

Sub $u = 2-x$

$du = -dx$

$-du = dx$

$$\int \frac{dx}{2-x} = -\int \frac{du}{u}$$

$$= -\ln|u| + C_1$$

$$= -\ln|2-x| + C_1$$

$$\ln|y| = -\ln|2-x| + C_1$$

$$e^{\ln|y|} = e^{-\ln|2-x| + C_1}$$

$$e^{\ln|y|} = e^{-\ln|2-x|} \cdot e^{C_1}$$

$$e^{\ln|y|} = e^{\ln|2-x|^{-1}} \cdot e^{C_1}$$

$$|y| = |2-x|^{-1} \cdot e^{C_1}$$

$$y = \frac{\pm e^{C_1}}{2-x}$$

$$\boxed{y = \frac{C}{2-x}}$$

$y=6$
$x=0$: $6 = \frac{C}{2}$

$$12 = C$$

$$\boxed{y = \frac{12}{2-x}}$$

Interval of solution?
Largest set of x-values on which y is continuous. Must contain the initial x-value, if one was given.

$$2-x \neq 0$$

$$2 \neq x$$

$$x<2 \quad \text{or} \quad x>2$$

But $x=0$ was given.
Interval of solution: $x<2$

Ex: Solve $x\,dx + y\,dy = 0$ with $y(2) = -1$.

Separable DE

$$x\,dx = -y\,dy$$

$$\int x\,dx = \int -y\,dy$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + C_1$$

$$x^2 = -y^2 + \cancel{2C_1}\, C$$

$$\boxed{y^2 = C - x^2}$$

Sub $y=-1$, $x=2$:

$$1 = C - 4$$

$$5 = C$$

$$\boxed{y^2 = 5 - x^2}$$

$$y = \pm\sqrt{5-x^2}$$

But $y(2) = -1$, choose $-$

$$\boxed{y = -\sqrt{5-x^2}}$$