List of HW Problems and Full Solutions on Website

Problems are on D2L

or www.leahhoward.com/252CP.pdf

Do Suggested HW for Section 1.1

1.1 Introduction Cont'd

Ex: Confirm $y = C_1 \cos x + C_2 \sin x$

solves $y'' + y = 0$.

Note: C_1 and C_2 are called parameters

C_1, C_2: any real numbers

Solution

$$y = C_1 \cos x + C_2 \sin x$$

$$y' = -C_1 \sin x + C_2 \cos x$$

$$y'' = -C_1 \cos x - C_2 \sin x$$

$$\begin{aligned}
\text{LS of DE} &= y'' + y \\
&= -C_1 \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x \\
&= 0 \\
&= \text{RS of DE} \checkmark
\end{aligned}$$

Terminology:

$y = C_1 \cos x + C_2 \sin x$ is a "2-parameter family of solutions"

Some <u>particular solutions</u>:

$y = 0$

$y = 8\cos x$

$y = -4\sin x$

$y = \sqrt{2}\cos x - \pi \sin x$

<u>Ex</u>: Confirm that $x = Ct^4$ is a solution of $\underbrace{t\frac{dx}{dt} - 4x = 0}_{\text{DE}}$.

solution

Solution $x = Ct^4$ (C: any real #)

$$\frac{dx}{dt} = 4Ct^3$$

$$\begin{aligned}
\text{LS of DE} &= t\frac{dx}{dt} - 4x \\
&= t(4Ct^3) - 4(Ct^4) \\
&= 4Ct^4 - 4Ct^4 \\
&= 0 \\
&= \text{RS of DE} \checkmark
\end{aligned}$$

Terminology:

$x = Ct^4$ is a "1-parameter family of solutions"

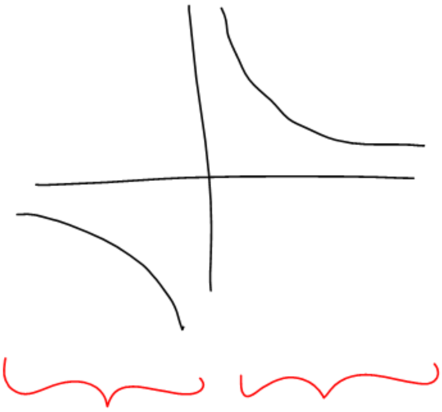
Interval of Solution:

Largest interval of x-values on which the solution y is continuous*

There may be several possible intervals.

* can be traced without lifting your pencil (no holes, no jumps, no asymptotes)

Ex: Solution $y = \frac{1}{x}$



Solution is continuous on $-\infty < x < 0$ and $0 < x < \infty$.

Intervals of solution: $-\infty < x < 0$ or $0 < x < \infty$.

REVIEW

$f(x)$	$f'(x)$
e^{2x}	$2e^{2x}$
$\ln x$	$\frac{1}{x}$
$\sin 2x$	$2\cos 2x$
$\cos 2x$	$-2\sin 2x$
$\tan 4x$	$4\sec^2 4x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\frac{2}{x^7} = 2x^{-7}$	$-14x^{-8}$
$\sqrt[3]{x} = x^{1/3}$	$\frac{1}{3}x^{-2/3}$

REVIEW

$f(x)$	$\int f(x)\,dx$
e^{7x}	$\frac{e^{7x}}{7} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\sin 2x$	$-\frac{\cos 2x}{2} + C$
$\cos 3x$	$\frac{\sin 3x}{3} + C$
$\frac{2}{x^7} = 2x^{-7}$	$-\frac{1}{3}x^{-6} + C$
$\sqrt[3]{x} = x^{1/3}$	$\frac{3}{4}x^{4/3} + C$
$\frac{1}{\sqrt{4-x^2}}$	$\arcsin\frac{x}{2} + C$
$\frac{1}{4+x^2}$	$\frac{1}{2}\arctan\frac{x}{2} + C$
$\cot x$	$\ln\lvert\sin x\rvert + C$ or $-\ln\lvert\csc x\rvert + C$

See formula sheet

1.2 Initial Value Problems (IVP)

Ex: $y' = y$ has solution $y = Ce^{x}$.

Solve:

a) $y' = y$, $\boxed{y(0) = 3}$ } IVP

initial condition

Solution $y = Ce^x$

Sub $x=0$, $y=3$:

$3 = Ce^0$

$3 = C$

$$\boxed{y = 3e^x}$$

b) $y' = y$, $\boxed{y(1) = -3}$ } IVP

initial condition

Solution $y = Ce^x$

Sub $x=1$, $y=-3$:

$-3 = Ce$

$-3e^{-1} = C$

$y = -3e^{-1}e^x$

$$\boxed{y = -3e^{x-1}}$$

Ex: $y' + 2xy^2 = 0$ has solution

$$y = \frac{1}{x^2+C}.$$

Solve $\begin{cases} y' + 2xy^2 = 0 \\ y(0) = -\frac{1}{4} \end{cases}$

Find the interval of solution.

Solution $y = \dfrac{1}{x^2+C}$

Sub $x=0$, $y = -\frac{1}{4}$: $\quad -\dfrac{1}{4} = \dfrac{1}{C}$

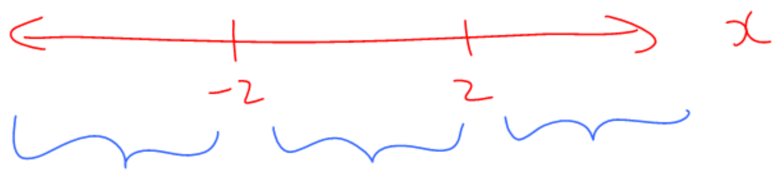
$$-4 = C$$

$$\boxed{y = \frac{1}{x^2-4}}$$

Interval of solution: $\quad x^2 - 4 \neq 0$

$$x^2 \neq 4$$

$$x \neq \pm 2$$



Possible intervals of solution.

Note: interval of solution must contain $x=0$ (initial condition)

Interval of solution: $-2 < x < 2$