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→ Math 252

1.1 Introduction

Differential equation (DE):

An equation involving derivatives.

e.g. $$\frac{dy}{dx} = 25 + y^2$$

$$x^2 y'' - 2xy' + 6y = 0$$

<u>Ex</u>: Confirm that $\overbrace{y = 5\tan 5x}^{\text{solution}}$ is a solution to $\underbrace{\frac{dy}{dx} = 25 + y^2}_{\text{DE}}$.

Solution $\begin{cases} y = 5\tan 5x \\ \frac{dy}{dx} = 25\sec^2 5x \end{cases}$

$$\text{LS of DE} = \frac{dy}{dx}$$

$$= 25\sec^2 5x$$

$$\text{RS of DE} = 25 + y^2$$

$$= 25 + [5\tan 5x]^2$$

$$= 25 + 25\tan^2 5x$$

$$= 25(1+\tan^2 5x)$$

$$= 25\sec^2 5x$$

$$= \text{LS of DE} \checkmark$$

<u>Ex</u>: Let P: population

t: time

Confirm that $P = \overbrace{\frac{6}{5}(1-e^{-20t})}^{\text{solution}}$

solves $\underbrace{\frac{dP}{dt} + 20P = 24}_{\text{DE}}$

Solution
$$\begin{cases} P = \frac{6}{5}(1-e^{-20t}) \\ \frac{dP}{dt} = \frac{6}{5}(20e^{-20t}) \\ \quad = 24e^{-20t} \end{cases}$$

$$\text{LS of DE} = \frac{dP}{dt} + 20P$$

$$= 24e^{-20t} + \underbrace{(20)\frac{6}{5}}_{24}(1-e^{-20t})$$

$$= 24e^{-20t} + 24 - 24e^{-20t}$$

$$= 24$$

$$= \text{RS of DE} \checkmark$$

Ex: Show that $x^2+y^2=25$ Solution

is a solution to $\underbrace{\frac{dy}{dx} = \frac{-x}{y}}_{\text{DE}}$.

Solution $x^2+y^2=25$

Implicit Differentiation (Take $\frac{d}{dx}$)

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\begin{aligned} \text{LS of DE} &= \frac{dy}{dx} \\ &= -\frac{x}{y} \\ &= \text{RS of DE} \end{aligned}$$

Notation for Derivatives:

$y', y'', y''', y^{(4)}, y^{(5)}, \ldots$

$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots$

Order of a DE:

The order of the highest derivative.

Ex: State the order of the DE

a) $y'' + 5(y')^3 + y = 0$

2nd order

b) $x^6 y''' - y = 0$

3rd order

c) $\frac{dy}{dx} = 25 + y^2$

1st order

Normal form of a DE:

Isolated the highest derivative.

Ex: DE: $5x y'' + y = x$

$$5x y'' = x - y$$

$$y'' = \frac{x - y}{5x} \quad \text{normal form}$$

or $$y'' = \frac{-1}{5x} y + \frac{1}{5}$$

normal form

Linear DE: $y, y', y'', \dots$ all to 1^{st} power

Coefficients don't involve y.

Ex: a) $3y'' + 2y = 0$

or $3y'' + 0y' + 2y = 0$

Linear DE

b) $8y' + e^{x} y = 0$

Linear DE

c) $(\sin y)\, y' + 10y = 0$

Non-linear DE

d) $(\sin x)\, y' + 10y = 0$

Linear DE

e) $y^2 = y''$

Non-linear DE