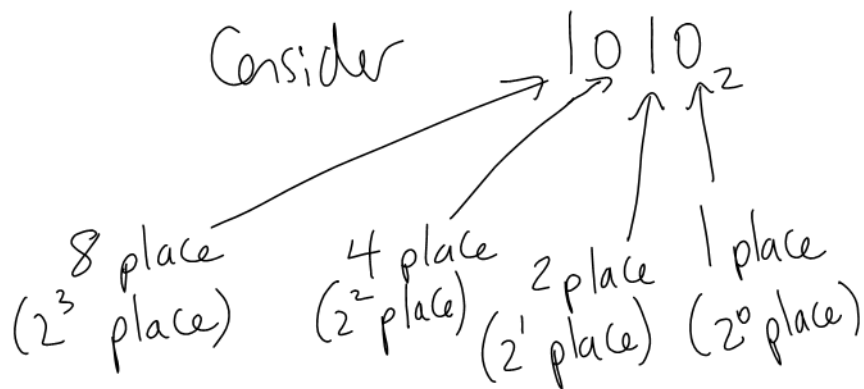


1.2 Binary and Hexadecimal

Binary: base 2

Allowed digits are 0, 1.

These are called "bits" or "binary digits".



Decimal	Binary
0	0 ₂
1	1 ₂
2	10 ₂
3	11 ₂
4	100 ₂
⋮	⋮
7	111 ₂
8	1000 ₂
9	1001 ₂

Ex: Convert to decimal

a) 1100₂ = 1 × 2³ + 1 × 2²

2³ place 2² place

$$= 1 \times 8 + 1 \times 4$$

$$= 12$$

b) $101110_2 = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1$

2^5 place $\nearrow \nearrow \nearrow \nearrow \nearrow$ 2^0 place

$$= 1 \times 32 + 1 \times 8 + 1 \times 4 + 1 \times 2$$

$$= 46$$

Hexadecimal: base 16

Allowed digits: 0, 1, ..., 9, A, B, C, D, E, F

Decimal	Hexadecimal
0	0_{16}
1	1_{16}
...	...
9	9_{16}
10	A_{16}
11	B_{16}
12	C_{16}
13	D_{16}
14	E_{16}
15	F_{16}
16	10_{16}
17	11_{16}

$\leftarrow$ 1 sixteen and 0 ones

$\leftarrow$ 1 sixteen and 1 one

4096 place
(16^3 place)

16 place
(16^1 place)

$\rightarrow$ BA98₁₆

$\leftarrow$ 1 place (16^0 place)

Ex: Convert to decimal

a) $2B_{16} = 2 \times 16^1 + 11 \times 16^0$

16^1 place 16^0 place

$$\begin{array}{l} A_{16} = 10 \\ B_{16} = 11 \end{array}$$

$$\begin{aligned} &= 2 \times 16 + 11 \times 1 \\ &= 43 \end{aligned}$$

b) $98003_{16} = 9 \times 16^4 + 8 \times 16^3 + 3 \times 16^0$

16^4 place 16^0 place

$$\begin{aligned} &= 9 \times 65536 + 8 \times 4096 + 3 \times 1 \\ &= 622\ 595 \end{aligned}$$

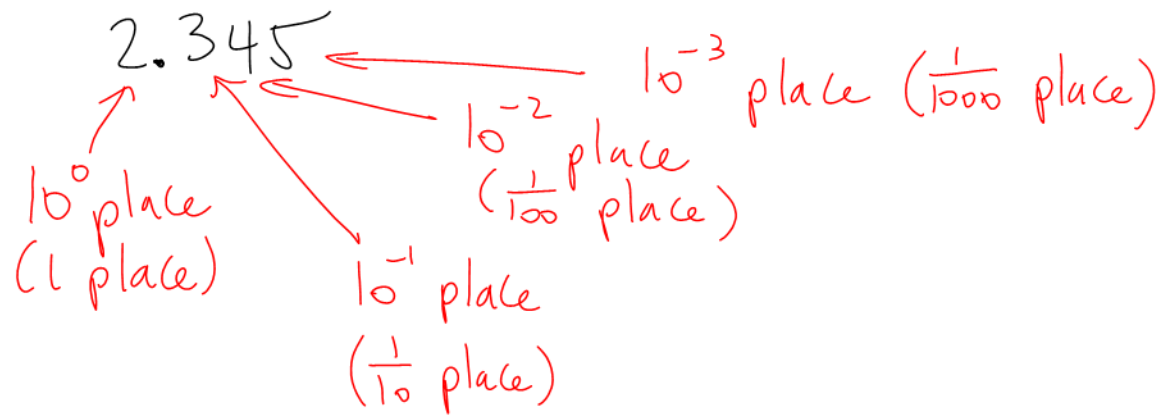
c) $B055_{16} = 11 \times 16^3 + 5 \times 16^1 + 5 \times 16^0$

16^3 place 16^0 place

$$\begin{array}{l} A_{16} = 10 \\ B_{16} = 11 \end{array}$$

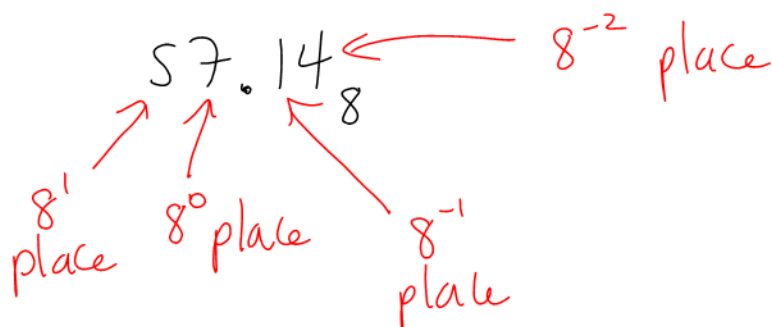
$$= 45\ 141$$

1.3 Converting Non-Integer Numbers to Decimal



$$\begin{aligned} 2.345 &= 2 \times 10^0 + 3 \times 10^{-1} + 4 \times 10^{-2} + 5 \times 10^{-3} \\ &= 2 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100} + 5 \times \frac{1}{1000} \end{aligned}$$

In other bases:



$$\begin{aligned} &= 5 \times 8^1 + 7 \times 8^0 + 1 \times 8^{-1} + 4 \times 8^{-2} \\ &= 5 \times 8 + 7 \times 1 + 1 \times \frac{1}{8} + 4 \times \frac{1}{64} \\ &= 47.1875 \end{aligned}$$

The dot is called the radix point rather than the decimal point.

Notice: The number to the left of the radix point is in the 1 place (associated with the exponent 0).

Ex: Convert to decimal

a) 11.011_2

2^1 place 2^0 place 2^{-1} place 2^{-2} place 2^{-3} place

$$\begin{aligned} &= 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-2} + 1 \times 2^{-3} \\ &= 1 \times 2 + 1 \times 1 + 1 \times \frac{1}{4} + 1 \times \frac{1}{8} \\ &= 3.375 \end{aligned}$$

b) $A0.3F6_{16}$ (round to 3 decimal places)

16^1 place 16^0 place 16^{-1} place 16^{-2} place 16^{-3} place

$$A_{16} = 10$$

$$F_{16} = 15$$

$$\begin{aligned} &= 10 \times 16^1 + 0 \times 16^0 + 3 \times 16^{-1} + 15 \times 16^{-2} + 6 \times 16^{-3} \\ &= 10 \times 16 + 3 \times \frac{1}{16} + 15 \times \frac{1}{256} + 6 \times \frac{1}{4096} \\ &\approx 160.248 \end{aligned}$$