Solutions

① $\mu = np = 1.5 \qquad \sigma = \sqrt{npq} \qquad q = 1-p$

$$= \sqrt{(6(0.25)(0.75))}$$

$$\approx 1.06$$

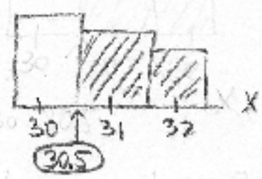
② $\boxed{np > 5 \quad \text{and } nq > 5}$ where $q = 1-p$

$120(0.08) > 5$ ✓ $\qquad 120(0.92) > 5$ ✓

③ Since $np > 5$ and $nq > 5$,

$P(X > 30)$ binomial

$\approx P(X > 30.5)$ normal



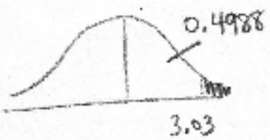
$\mu = np = 21.6 \qquad \sigma = \sqrt{npq} \approx 2.939$

$q = 1-p = 0.4$

$$z = \frac{30.5-\mu}{\sigma} \approx 3.03$$

$= P(z > 3.03)$

$=$



$= 0.5 - 0.4988$

$= 0.0012$

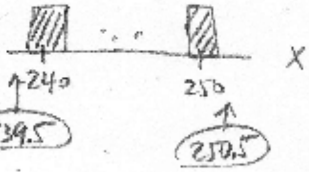
The probability is approximately 0.0012

(4) Binomial $n = 500$ $\quad p = P(\text{odd}) = \frac{1}{2}$ $\quad q = 1 - p = \frac{1}{2}$

Find $P(240 \le X \le 250)$ binomial

Since $np > 5$ and $nq > 5$

$\approx P(239.5 \le X \le 250.5)$ normal

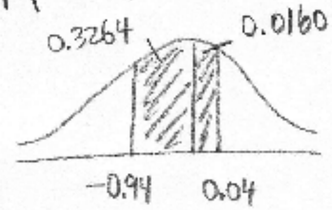


$\mu = np = 250 \qquad \sigma = \sqrt{npq} \approx 11.180$

$z_1 = \dfrac{239.5 - \mu}{\sigma} \approx -0.94 \qquad z_2 = \dfrac{250.5 - \mu}{\sigma} \approx 0.04$

$= P(-0.94 \le z \le 0.04)$

$=$



$= 0.3264 + 0.0160$

$= 0.3424$

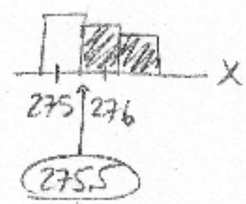
The probability is approximately 0.3424

(5) Binomial $n = 500$ $p = P(\text{odd}) = \frac{1}{2}$ $q = 1 - p = \frac{1}{2}$

Find $P(X > 275)$ binomial

Since $np > 5$ and $nq > 5$

$\approx P(X > 275.5)$ normal



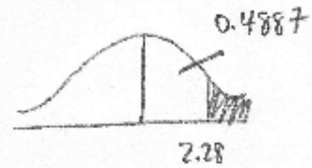
$\mu = np = 250$ $\sigma = \sqrt{npq}$

≈ 11.180

$$z = \frac{275.5 - \mu}{\sigma} \approx 2.28$$

$= P(z > 2.28)$

$=$



$= 0.5 - 0.4887$

$= 0.0113$

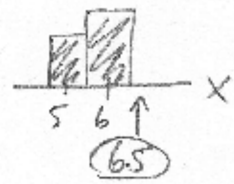
The probability is approximately 0.0113

⑥ Binomial $n=30$ $p=P(\heartsuit)=\frac{1}{4}$ $q=1-p=\frac{3}{4}$

Find $P(X \le 6)$ binomial

Since $np > 5$ and $nq > 5$

$\approx P(X \le 6.5)$ normal

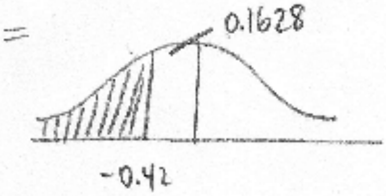


$\mu = np = 7.5$ $\sigma = \sqrt{npq} \approx 2.372$

$z = \dfrac{6.5-\mu}{\sigma} \approx -0.42$

$= P(z \le -0.42)$

$=$



$= 0.5 - 0.1628$

$= 0.3372$

The probability is approximately 0.3372

⑦ With $n=30$ $p=\frac{1}{4}$ $q=\frac{3}{4}$

$P(X \le 6)$ binomial

$= P(X=0) + P(X=1) + \dots + P(X=6)$

$= 30C0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{30} + 30C1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{29} + \dots + 30C6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{24}$

≈ 0.3481

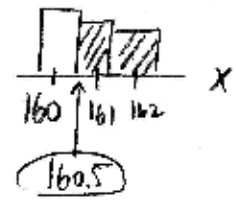
Fairly close to the answer in Question 6.

(8) Binomial $n=300$ $p=P(H)=\frac{1}{2}$ $q=1-p=\frac{1}{2}$

Find $P(X>160)$ binomial

Since $np>5$ and $nq>5$

$\approx P(X>160.5)$ normal

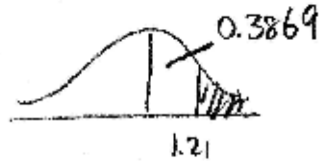


$\mu = np = 150$ $\sigma = \sqrt{npq}$
≈ 8.660

$$z = \frac{160.5-\mu}{\sigma} \approx 1.21$$

$= P(z>1.21)$

$=$



$= 0.5 - 0.3869$

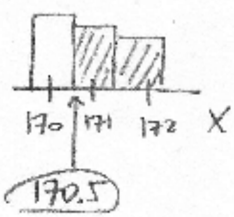
≈ 0.1131

The probability is approximately 0.1131

(9) Binomial $n=300$ $p=P(H)=\frac{1}{2}$ $q=1-p=\frac{1}{2}$

Find $P(X>170)$ binomial

Since $np>5$ and $nq>5$



$\approx P(X>170.5)$ normal

$\mu=np=150$ $\sigma=\sqrt{npq}$

≈ 8.660

$z=\frac{170.5-\mu}{\sigma}\approx 2.37$

$= P(z>2.37)$

$=$

$= 0.5-0.4911$

$= 0.0089$

The probability is approximately 0.0089

(10) Let X = # boxes that arrive on time (successes)

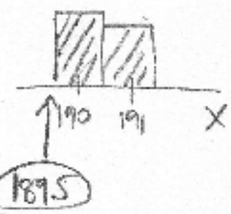
X is binomial with $n=250$ (# of trials)

$q = P(\text{delayed}) = 0.3$

$p = P(\text{on time}) = 0.7$

Find $P(X \geq 190)$ binomial

Since $np > 5$ and $nq > 5$

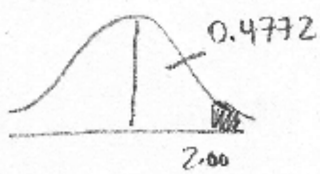


$\approx P(X \geq 189.5)$ normal

$\mu = np = 175$ $\sigma = \sqrt{npq} \approx 7.246$

$z = \dfrac{189.5 - \mu}{\sigma} \approx 2.00$

$= P(z \geq 2.00)$

$=$ 

$= 0.5 - 0.4772$

$= 0.0228$

The approximate probability that they meet the demand (get ≥ 190 boxes on time) is 0.0228