Math 251 X02
Assignment 1: MATLAB

Name: _____

Due: At the beginning of class, Date ???
Total: 25 marks

INSTRUCTIONS
* Submit your work on this paper.
* If you are away on the duedate then submit via the D2L Dropbox.
* You may discuss with others but your write-up must be your own work.
* Show all your work for full marks.

1. [5 marks] Given $A = \begin{bmatrix} 1 & 3 \\ 1 & -4 \\ 1 & 7 \\ 1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 68 \\ 126 \\ 34 \\ 160 \end{bmatrix}$,

we want to compute $(A^T A)^{-1} A^T \mathbf{b}$.

a) Write down the MATLAB commands for inputting A, inputting $\mathbf{b}$, and calculating $(A^T A)^{-1} A^T \mathbf{b}$.

```
>> format short
>> A = [1  3; 1  -4; 1  7; 1  -8];
>> b = [68; 126; 34; 160];
>> inv(A' * A) * A' * b
```

b) Perform the calculation in Octave Online. Then write down your answer, rounding your vector components to two decimal places.

$$\begin{bmatrix} 92.81 \\ -8.38 \end{bmatrix}$$

2. [5 marks] Given $\mathbf{a} = \begin{bmatrix} 7 \\ -3 \\ 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 21 \\ -36 \\ 5 \end{bmatrix}$, we want to compute $\mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$.

a) Write down the MATLAB commands for inputting $\mathbf{a}$, inputting $\mathbf{b}$, and calculating $\mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$.

```
>> format rat
>> a = [7  -3  9];
>> b = [21  -36  5];
>> b - (dot(a,b)/norm(a)^2)*a
```

ALTERNATIVELY the last line could be

```
>> b - (dot(a,b)/dot(a,a))*a
```

b) Perform the calculation in Octave Online. Then write down your answer, using exact values.

$$\left[\frac{819}{139}, \frac{-4104}{139}, \frac{-2005}{139} \right]$$

Note: Vectors could also be written as column vectors.

3. [4 marks] Let a, b, c, d, u and v be real numbers. If $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \neq 0$ then the values of x and y that solve

$$\begin{aligned} ax + by &= u \\ cx + dy &= v \end{aligned}$$

$$\text{are } x = \frac{\det \begin{bmatrix} u & b \\ v & d \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}} \text{ and } y = \frac{\det \begin{bmatrix} a & u \\ c & v \end{bmatrix}}{\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}}.$$

We want to use these formulas to find the values of x and y that solve

$$\begin{aligned} -19x + 6y &= \frac{1797}{7} \\ 25x + 8y &= \frac{2809}{21} \end{aligned}$$

a) Write down the MATLAB commands for inputting the appropriate matrices and calculating the exact values of x and y.

(Let's call the matrices A1, A2, A).

```
>> format rat
>> A = [-19   6; 25   8];
>> A1 = [1797/7   6; 2809/21   8];
>> A2 = [-19   1797/7; 25   2809/21];
>> x = det(A1)/det(A)
>> y = det(A2)/det(A)
```

b) Perform the calculation in Octave Online. Then write down the exact values of x and y.

$x = -\frac{29}{7}$ $\qquad$ $y = \frac{89}{3}$

4. [5 marks] We want to find all curves $y = c_0 + c_1x + c_2x^2 + c_3x^3$ that pass through both the points $(2, 9)$ and $(3, 8)$.

a) Write down the augmented matrix for the appropriate system of equations.

$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3$

Sub $x=2, y=9$: $9 = c_0 + 2c_1 + 4c_2 + 8c_3$

$x=3, y=8$: $8 = c_0 + 3c_1 + 9c_2 + 27c_3$

$$\begin{array}{c} c_0 \quad c_1 \quad c_2 \quad c_3 \\ \left[\begin{array}{cccc|c} 1 & 2 & 4 & 8 & 9 \\ 1 & 3 & 9 & 27 & 8 \end{array}\right] \end{array}$$

b) Calculate the RREF of the augmented matrix in Octave Online. Write the RREF below. Then find all the possible values of c_0, c_1, c_2 and c_3 by hand.

$$\begin{array}{c} c_0 \quad c_1 \quad c_2 \quad c_3 \\ \left[\begin{array}{cccc|c} 1 & 0 & -6 & -30 & 11 \\ 0 & 1 & 5 & 19 & -1 \end{array}\right] \end{array}$$

$c_2 = s$ $\quad$ $c_3 = t$

$c_0 - 6c_2 - 30c_3 = 11 \Rightarrow \quad c_0 = 11 + 6s + 30t$

$c_1 + 5c_2 + 19c_3 = -1 \Rightarrow \quad c_1 = -1 - 5s - 19t$

5. [6 marks] We want to balance $S + HNO_3 \to H_2SO_4 + NO_2 + H_2O$.

a) Write down the augmented matrix for the appropriate system of equations.

$v(S) + w(HNO_3) \to x(H_2SO_4) + y(NO_2) + z(H_2O)$

S: $v = x \Rightarrow v - x = 0$

H: $w = 2x + 2z \Rightarrow w - 2x - 2z = 0$

N: $w = y \Rightarrow w - y = 0$

O: $3w = 4x + 2y + z \Rightarrow 3w - 4x - 2y - z = 0$

$$\left[\begin{array}{ccccc|c} v & w & x & y & z & \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 3 & -4 & -2 & -1 & 0 \end{array}\right]$$

b) Calculate the RREF of the augmented matrix in Octave Online. Write the RREF below. Then finish balancing the chemical equation by hand.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \end{array}\right]$$

$z = t$

$v = \frac{1}{2}t$, $w = 3t$, $x = \frac{1}{2}t$, $y = 3t$

We want the smallest positive integer solution $\Rightarrow t = 2$.

$[v, w, x, y, z] = [1, 6, 1, 6, 2]$

$S + 6HNO_3 \to H_2SO_4 + 6NO_2 + 2H_2O$