

Math 251 X02
Test Three

Time: 50 minutes
Total: 25 marks

Name: _____

1. [4 marks] Find all the eigenvalues of $A = \begin{bmatrix} 7 & -2 \\ -1 & 6 \end{bmatrix}$.

2. [6 marks] A and B are 3×3 matrices with $\det A = -2$ and $\det B = 7$. State the determinant of the following matrices or write “not enough information”.

a) A^{-1}

b) A^T

c) $A + B$

d) AB

e) A^4

f) $2B$

3. [4 marks] $A = \begin{bmatrix} 4 & 8 & 13 & 18 \\ 3 & 6 & 10 & 10 \\ 2 & 4 & 6 & 8 \end{bmatrix}$ has RREF = $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Find a basis for:

a) the row space of A

b) the column space of A

c) the null space of A

4. [5 marks] Find the standard matrix for:

a) the transformation T where $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -4x + 7y \\ x - 2y \end{bmatrix}$

b) the transformation S where S rotates a vector in \mathbb{R}^2 by 60°

c) the transformation T followed by S

5. [6 marks] Solve:

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 6 & 1 & 3 \\ 0 & 2 & -2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -41 \\ 121 \\ -76 \end{bmatrix}$$