

Math 251 X02
Test Three

Time: 50 minutes
Total: 25 marks

Name: _____

1. [4 marks] $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 1 & 0 & 1 \\ 1 & 3 & 3 & 5 \end{bmatrix}$ has RREF $= \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Find a basis for:

a) the column space of A

b) the row space of A

c) the null space of A

2. [5 marks] Let $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -3x + 8y \\ 2x - 7y \end{bmatrix}$.

Let R be the transformation that rotates a vector in $\mathbb{R}^2$ by 30° clockwise. Find the standard matrix for the transformation that first performs R and then T . Simplify your answer to a single matrix.

3. [5 marks] Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ -8 & 2 & 5 \end{bmatrix}$. Find the LU factorization of A.

4. [5 marks] Use Cramer's Rule to find y . Show all your work.
(You do not need to find x or z).

$$\begin{aligned}2x - 3y + 4z &= 53 \\2x + y + 2z &= 85 \\-2x + 2y - 2z &= -34\end{aligned}$$

5. [6 marks] a) Find all the eigenvalues of $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$.

b) Find one eigenvector of $A = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$ corresponding to $\lambda = 4$.