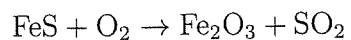


1. [3 marks] Write down the system of equations you would solve to balance the following chemical equation. Do not solve the system.



$$\text{Fe:} \quad w = 2y$$

$$\text{S:} \quad w = z$$

$$\text{O:} \quad 2x = 3y + 2z$$

OR

$$\begin{array}{cccc|c} w & x & y & z & \\ \hline 1 & 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}$$

2. [5 marks] If possible, write $\mathbf{w} = \begin{bmatrix} 51 \\ 183 \\ 267 \end{bmatrix}$ as a linear combination of

$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$. Show all your work.

$$\text{Let } c_1 \vec{u}_1 + c_2 \vec{u}_2 = \vec{w}$$

$$\begin{array}{cc|c} c_1 & c_2 & \\ \hline 1 & 2 & 51 \\ -2 & 1 & 183 \\ 3 & 8 & 267 \end{array}$$

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 - 3R_1 \end{array} \begin{array}{cc|c} \hline 1 & 2 & 51 \\ 0 & 5 & 285 \\ 0 & 2 & 114 \\ \hline \end{array}$$

$$\frac{R_2}{5} \begin{array}{cc|c} \hline 1 & 2 & 51 \\ 0 & 1 & 57 \\ 0 & 2 & 114 \\ \hline \end{array}$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 2R_2 \end{array} \begin{array}{cc|c} \hline 1 & 0 & -63 \\ 0 & 1 & 57 \\ 0 & 0 & 0 \\ \hline \end{array}$$

$$c_1 = -63$$

$$c_2 = 57$$

$$\vec{w} = -63\vec{u}_1 + 57\vec{u}_2$$

3. [6 marks] Let $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -6 \\ 8 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 \\ -5 & 1 \end{bmatrix}$.

Compute $(A + 7I)^T - BC$.

$$\begin{aligned} A + 7I &= \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -3 \\ 1 & 11 \end{bmatrix} \end{aligned}$$

$$(A + 7I)^T = \begin{bmatrix} 9 & 1 \\ -3 & 11 \end{bmatrix}$$

$$\begin{aligned} BC &= \begin{bmatrix} 1 & -6 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -5 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 33 & -2 \\ 4 & 36 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (A + 7I)^T - BC &= \begin{bmatrix} 9 & 1 \\ -3 & 11 \end{bmatrix} - \begin{bmatrix} 33 & -2 \\ 4 & 36 \end{bmatrix} \\ &= \begin{bmatrix} -24 & 3 \\ -7 & -25 \end{bmatrix} \end{aligned}$$

4. [6 marks] a) Find A^{-1} for $A = \begin{bmatrix} 7 & -2 \\ 3 & 5 \end{bmatrix}$.

$$\det A = 7(5) - (-2)(3) \\ = 41$$

$$A^{-1} = \frac{1}{41} \begin{bmatrix} 5 & 2 \\ -3 & 7 \end{bmatrix}$$

b) Solve the system below. Your answer will involve the constant c .

$$7x - 2y = 157$$

$$3x + 5y = c$$

$$\vec{x} = A^{-1} \vec{b}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{41} \begin{bmatrix} 5 & 2 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} 157 \\ c \end{bmatrix}$$

$$= \frac{1}{41} \begin{bmatrix} 785 + 2c \\ -471 + 7c \end{bmatrix}$$

5. [5 marks] Find the general form of $\text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}\right)$.

$$\text{Let } c_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Each zero row of the REF will give a condition on x, y, z .

$$\begin{array}{cc|c} c_1 & c_2 & \\ \hline 1 & 6 & x \\ 2 & 14 & y \\ 4 & 30 & z \end{array}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \begin{array}{cc|c} 1 & 6 & x \\ \hline 0 & 2 & y - 2x \\ 0 & 6 & z - 4x \end{array}$$

$$\frac{R_2}{2} \begin{array}{cc|c} 1 & 6 & x \\ \hline 0 & 1 & \frac{y-2x}{2} \\ 0 & 6 & z - 4x \end{array}$$

$$R_3 - 6R_2 \begin{array}{cc|c} 1 & 6 & x \\ \hline 0 & 1 & \frac{y-2x}{2} \\ 0 & 0 & 2x - 3y + z \end{array} \text{ REF}$$

$$\begin{aligned} & \leftarrow z - 4x - 6\left(\frac{y-2x}{2}\right) \\ & = z - 4x - 3(y - 2x) \\ & = z - 4x - 3y + 6x \\ & = 2x - 3y + z \end{aligned}$$

$$\text{Consistent system} \Rightarrow \begin{aligned} 2x - 3y + z &= 0 \\ z &= -2x + 3y \end{aligned}$$

The span is $\left\{ \begin{bmatrix} x \\ y \\ -2x + 3y \end{bmatrix} \right\}$