

## Math 251 X02 Assignment 2

Name: \_\_\_\_\_

This assignment has 5 questions.

Covers Sections: 4.3-4.4 and 5.1-5.3

Due: At the beginning of class, Monday Dec. 2

### INSTRUCTIONS:

Show all your work for full marks.

You may discuss with others but you may not copy another person's work.

If you are away on the due date then you can submit to the D2L Dropbox.

1. [3 marks]  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is an orthogonal basis for  $\mathbb{R}^3$ , where:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \text{ and } \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}.$$

Write  $\mathbf{w} = \begin{bmatrix} 93 \\ -61 \\ 87 \end{bmatrix}$  as a linear combination of the basis vectors.

2. [7 marks] Find an orthogonal basis for  $\text{span}\left(\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}\right)$ .

3. [6 marks] Let  $W = \text{span}\left(\begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 7 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -11 \\ 2 \\ -1 \end{bmatrix}\right)$ .

Find a basis for  $W^\perp$ .

4. [7 marks] The matrix  $A$  has eigenvalue  $\lambda_1 = 2$  corresponding to the eigenvector  $\vec{x}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$  and eigenvalue  $\lambda_2 = 3$  corresponding to the eigenvector  $\vec{x}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ . Find  $A^3 \begin{bmatrix} 22 \\ 23 \end{bmatrix}$  using the formula  $A^n(c_1\vec{x}_1 + c_2\vec{x}_2) = c_1\lambda_1^n\vec{x}_1 + c_2\lambda_2^n\vec{x}_2$ .

5. [7 marks] The matrix  $A$  has the eigenvalue 2 corresponding to the eigenvector  $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$  and the eigenvalue 3 corresponding to the eigenvector  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ . Find  $A^n$ . Simplify your answer as much as possible.