(1) Solve $\underbrace{L U \bar{x}}_{\bar{y}} = \bar{b}$

First solve $L\bar{y} = \bar{b}$, then solve $U\bar{x} = \bar{y}$.

$L\bar{y} = \bar{b}$:
$$\begin{array}{ccc|c} y_1 & y_2 & y_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 58 \\ -3 & 1 & 0 & -156 \\ 7 & 9 & 1 & 632 \end{array}\right]$$

$y_1 = 58$

$-3y_1 + y_2 = -156 \Rightarrow -174 + y_2 = -156 \Rightarrow y_2 = 18$

$7y_1 + 9y_2 + y_3 = 632 \Rightarrow 406 + 162 + y_3 = 632 \Rightarrow y_3 = 64$

$U\bar{x} = \bar{y}$:
$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & 58 \\ 0 & 5 & 6 & 18 \\ 0 & 0 & 8 & 64 \end{array}\right]$$

$8x_3 = 64 \Rightarrow x_3 = 8$

$5x_2 + 6x_3 = 18 \Rightarrow 5x_2 + 48 = 18 \Rightarrow x_2 = -6$

$2x_1 - 3x_2 + 4x_3 = 58 \Rightarrow 2x_1 + 18 + 32 = 58 \Rightarrow x_1 = 4$

$$\bar{x} = \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix}$$

(2) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix}$

and find a basis for row(A).

$$R_2 - R_1 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$R_3 + 2R_2 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \text{REF}$$

To find a basis for row(A), use the nonzero rows of the REF/RREF of A.

$$\mathcal{B} = \{ [1 \ 1 \ 0], [0, -1, -1] \}$$

Note: If your REF was different then your answer will look slightly different.

③ Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & -1 & 2 \end{bmatrix}$

and find a basis for Col(A).

$$R_2 - R_1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix}$$

$$R_3 - R_2 \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}_{REF}$$

Use the pivots as a guide for which columns of A to use.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Note: If your REF was different then your answer will look slightly different.

(4) Solve $A\bar{x}=\vec{0}$. Each parameter produces a basis vector.

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -2 & -4 & -6 & 0 \\ 3 & 6 & 9 & 0 \end{array}\right]$$

R_2+2R_1

R_3-3R_1

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]_{RREF}$$

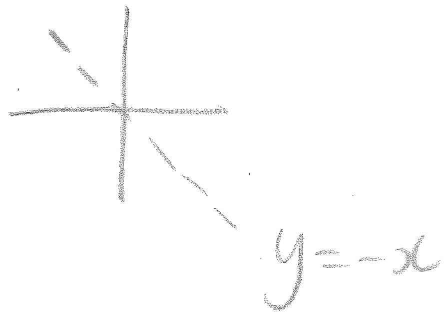
$x_2=s$ $\quad$ $x_3=t$

$$x_1+2x_2+3x_3=0 \Rightarrow x_1=-2s-3t$$

$$\bar{x}=\begin{bmatrix}-2\\1\\0\end{bmatrix}s+\begin{bmatrix}-3\\0\\1\end{bmatrix}t$$

$$\mathcal{B}=\left\{\begin{bmatrix}-2\\1\\0\end{bmatrix},\begin{bmatrix}-3\\0\\1\end{bmatrix}\right\}$$

(5)



$$S\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}0\\-1\end{bmatrix} \qquad S\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}-1\\0\end{bmatrix}$$

$$[S]=\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$$

$$[R]=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}_{\theta=-60^\circ}$$

$$=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\-\sqrt{3} & 1\end{bmatrix}$$

$$[R][S]=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\-\sqrt{3} & 1\end{bmatrix}\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}-\sqrt{3} & -1\\-1 & \sqrt{3}\end{bmatrix}$$