

Math 251 X02 Assignment 4

Covers: Sections 4.3-4.4 and 5.1-5.3

Due: Thurs Nov 27 at 11:30am

INSTRUCTIONS:

This assignment will be marked for completion.

Solutions will be posted on the course website 24 hours after the deadline.

You may not copy the work of another person or AI.

Submit jpg or pdf files to the D2L Dropbox.

1. $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for $\mathbb{R}^3$, where:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \text{ and } \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}. \text{ Write } \mathbf{w} = \begin{bmatrix} 93 \\ -61 \\ 87 \end{bmatrix}$$

as a linear combination of the basis vectors.

2. Find an orthogonal basis for $\text{span}\left(\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}\right)$.

3. Let $W = \text{span}\left(\begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 7 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -11 \\ 2 \\ -1 \end{bmatrix}\right)$.

Find a basis for $W^\perp$.

4. The matrix A has eigenvalue $\lambda_1 = 2$ corresponding to the

eigenvector $\vec{x}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and eigenvalue $\lambda_2 = 3$ corresponding to the eigenvector

$\vec{x}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Find $A^3 \begin{bmatrix} 22 \\ 23 \end{bmatrix}$ using the formula $A^n(c_1\vec{x}_1 + c_2\vec{x}_2) = c_1\lambda_1^n\vec{x}_1 + c_2\lambda_2^n\vec{x}_2$.

5. The matrix A has the eigenvalue 2 corresponding to the

eigenvector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and the eigenvalue 3 corresponding to the eigenvector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Find A^n . Simplify your answer as much as possible.