1. [5 marks] Find the equation of the tangent plane to $z = 4\ln(x^2+2) + 9x\sin(2y)$ at the point $(x, y) = (1, \pi)$.

$$z_x = \frac{4}{x^2+2}(2x) + 9\sin 2y$$

$$z_y = 18x\cos 2y$$

At $(x,y) = (1,\pi)$:

$$z_x = \frac{8}{3}$$

$$z_y = 18$$

$$z = 4\ln 3$$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= [-\frac{8}{3}, -18, 1]$$

$$\vec{n} \cdot \bar{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} -\frac{8}{3} \\ -18 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{8}{3} \\ -18 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \pi \\ 4\ln 3 \end{bmatrix}$$

$$-\frac{8}{3}x - 18y + z = -\frac{8}{3} - 18\pi + 4\ln 3$$

2. [4 marks] Use differentials to approximate $f(a+0.5, b-0.2)$ given $f(x,y) = 4(\sqrt{x^2+y^2})$ and $f(a,b) = c$. Your answer will involve a, b, and c.

$$f = 4\sqrt{x^2+y^2}$$

$$f_x = 4 \cdot \frac{1}{2}(x^2+y^2)^{-1/2}(2x) = \frac{4x}{\sqrt{x^2+y^2}}$$

$$f_y = 4 \cdot \frac{1}{2}(x^2+y^2)^{-1/2}(2y) = \frac{4y}{\sqrt{x^2+y^2}}$$

$$df = f_x\,dx + f_y\,dy$$

$$f(x+dx, y+dy) \approx f(x,y) + df$$

$$f(x+dx, y+dy) \approx f(x,y) + \frac{4x}{\sqrt{x^2+y^2}}dx + \frac{4y}{\sqrt{x^2+y^2}}dy$$

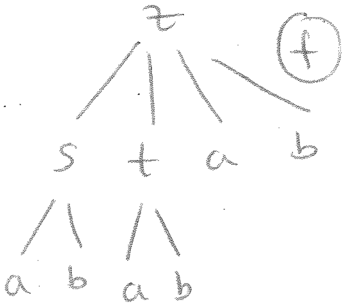
$$\boxed{\text{Sub } x=a,\ dx=0.5,\ y=b,\ dy=-0.2}$$

$$f(a+0.5, b-0.2) \approx \underbrace{\cancel{f(a,b)}}_{c} + \frac{4a}{\sqrt{a^2+b^2}}(0.5) + \frac{4b}{\sqrt{a^2+b^2}}(-0.2)$$

$$\approx c + 2\frac{a}{\sqrt{a^2+b^2}} - 0.8\frac{b}{\sqrt{a^2+b^2}}$$

3. [5 marks] Use the Multivariable Chain Rule to find z_b evaluated at $(a, b) = (1, 2)$ given:

$z = 2s^2 - 3s^3t^2 + 4t^2 + \frac{4b^3}{a}, \quad s = a^2 - 3b, \quad t = 2ab.$



$z_b = f_s s_b + f_t t_b + f_b$

$= (4s - 9s^2t^2)(-3) + (-6s^3t + 8t)(2a) + \frac{12b^2}{a}$

At $(a, b) = (1, 2)$: $s = -5$ $t = 4$

Sub $a = 1$, $b = 2$, $s = -5$, $t = 4$:

$z_b = (-3620)(-3) + (3032)(2) + 48$

$= 16972$

4. [6 marks] Find the absolute maximum of $z = x^4y^2$ over the region $x^2 + y^2 \leq 1$. Give the absolute maximum value and all points (x, y) where it is achieved.

1) Interior Critical Points

$$\left.\begin{aligned} z_x &= 4x^3y^2 \\ z_y &= 2x^4y \end{aligned}\right\} \text{ both 0 or undefined}$$

$\{(x,y)$ in $x^2+y^2 \leq 1$ such that $x=0$ or $y=0\}$

Note: A circle has 0 corners and 1 side

~~2) Corners~~

3) Side 1: $x^2+y^2=1$

$y^2 = 1-x^2 \rightarrow z = x^4y^2$

$z = x^4(1-x^2)$

$z = x^4 - x^6$

$z' = 4x^3 - 6x^5$

Set $z'=0$: $4x^3-6x^5=0$

$x^3(4-6x^2)=0$

$x=0 \qquad x^2 = \frac{4}{6} = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

$y^2 = 1-x^2$

$y^2 = \frac{1}{3}$

$y = \pm\frac{1}{\sqrt{3}}$

$\rightarrow$

Points	$z = x^4 y^2$
$x=0$ or $y=0$	0
$\left(\pm\sqrt{\frac{2}{3}}, \pm\frac{1}{\sqrt{3}}\right)$	$\frac{4}{27}$

The absolute maximum is $z = \frac{4}{27}$, achieved at $\left(\pm\sqrt{\frac{2}{3}}, \pm\frac{1}{\sqrt{3}}\right)$.