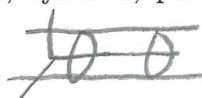


1. [3 marks] State whether the surface is a cone, cylinder, paraboloid, plane or sphere:

a) $x^2 + z^2 = 16$

cylinder



b) $z^2 = 6 - x^2 - y^2$

sphere



c) $z = 6 - x^2 - y^2$

paraboloid



2. [5 marks] Find the equation of the tangent plane to $z = 6x^4 - 2x^3 + y^2 - 7y$ at the point $(x, y, z) = (1, -1, 12)$.

$$z_x = 24x^3 - 6x^2, \quad z_y = 2y - 7$$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= [6x^2 - 24x^3, 7 - 2y, 1]$$

$$= [-18, 9, 1]$$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

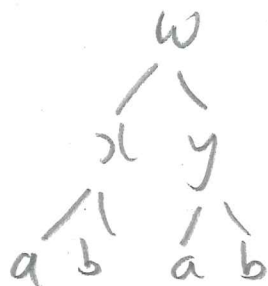
$$\begin{bmatrix} -18 \\ 9 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -18 \\ 9 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 12 \end{bmatrix}$$

$$-18x + 9y + z = -15$$

$$\text{or } 18x - 9y - z = 15$$

3. [6 marks] Use the Multivariable Chain Rule to find w_b at $(a, b) = (0, 0)$ given:

$$w = e^{xy} + \ln(x^2y + 1), \quad x = 2 + \sin(4a + 5b), \quad y = 2 + \cos(\pi - a - 6b).$$



$$w_b = w_x x_b + w_y y_b$$

$$= \left[ye^{xy} + \frac{2xy}{x^2y+1} \right] 5 \cos(4a+5b)$$

$$+ \left[xe^{xy} + \frac{x^2}{x^2y+1} \right] 6 \sin(\pi - a - 6b)$$

$$(a, b) = (0, 0) \Rightarrow (x, y) = (2, 1)$$

$$\text{Sub } a=0, b=0, x=2, y=1:$$

$$w_b = \left[e^2 + \frac{4}{5} \right] \underbrace{5 \cos 0}_1 + \left[2e^2 + \frac{4}{5} \right] \underbrace{6 \sin \pi}_0$$

$$= 5e^2 + 4$$

④ marks for differentiation

② marks for evaluation

4. [5 marks] Let $f = \frac{7(\sqrt{x})}{y^3}$

The maximum relative error in x is $\pm 2\%$.

The maximum relative error in y is $\pm 5\%$.

Estimate the maximum relative error in f .

$$f = 7x^{1/2}y^{-3}$$

$$f_x = \frac{7}{2}x^{-1/2}y^{-3}, \quad f_y = -21x^{1/2}y^{-4}$$

$$df = f_x dx + f_y dy$$

$$df = \frac{7}{2}x^{-1/2}y^{-3}dx - 21x^{1/2}y^{-4}dy$$

$$\frac{df}{f} = \frac{\frac{7}{2}x^{-1/2}y^{-3}dx}{7x^{1/2}y^{-3}} - \frac{21x^{1/2}y^{-4}dy}{7x^{1/2}y^{-3}}$$

$$\frac{df}{f} = \frac{1}{2} \frac{dx}{x} - 3 \frac{dy}{y}$$

To maximize $\frac{df}{f}$, use $\frac{dx}{x} = 2\%$ and $\frac{dy}{y} = -5\%$.

$$\begin{aligned} \left(\frac{df}{f}\right)_{\max} &= \frac{1}{2}(2\%) - 3(-5\%) \\ &= 16\% \end{aligned}$$

5. [6 marks] Find the absolute maximum of $z = x^6 y^2$ over the region $x^2 + y^2 \leq 4$.
Give the absolute maximum value and all points (x, y) where it is achieved.

1) Interior Critical Points

$$\left. \begin{aligned} z_x &= 6x^5 y^2 \\ z_y &= 2x^6 y \end{aligned} \right\} \text{ both 0 or undefined}$$

$$\Rightarrow x=0 \text{ or } y=0$$

$\{(x, y) \text{ in } x^2 + y^2 < 4 \text{ such that } x=0 \text{ or } y=0\}$

Note: A circle has 1 side and 0 corners.

2) Critical Points on side 1: $x^2 + y^2 = 4$

$$y^2 = 4 - x^2 \rightarrow z = x^6 y^2$$

$$z = x^6 (4 - x^2)$$

$$z = 4x^6 - x^8$$

$$z' = 24x^5 - 8x^7$$

$$\text{Set } z' = 0 : 24x^5 - 8x^7 = 0$$

$$8x^5 (3 - x^2) = 0$$

$$\downarrow$$

$$x=0$$

 (already covered)

$$\downarrow$$

$$3 - x^2 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$y^2 = 4 - x^2 = 1 \Rightarrow y = \pm 1$$

$$\rightarrow$$

Point	$z = x^6 y^2$
$x=0$ or $y=0$	0
$(\pm\sqrt{3}, \pm 1)$	27 $\leftarrow$ absolute maximum

The absolute maximum is $z=27$,
achieved at $(x,y) = (\pm\sqrt{3}, \pm 1)$.