1. [5 marks] Find the equation of the tangent plane to $z = \frac{5}{4}e^{3xy} + 8y\cos(3x) + \pi$ at the point $(x, y) = (0, -2)$.

$$z_x = \frac{15}{4} y e^{3xy} - 24y \sin 3x$$

$$z_y = \frac{15}{4} x e^{3xy} + 8\cos 3x$$

At $(x, y) = (0, -2)$: $z_x = \frac{-15}{2}$

$z_y = 8$

$z = \frac{5}{4} - 16 + \pi$

$= \pi - \frac{59}{4}$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= [\tfrac{15}{2}, -8, 1]$$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} \frac{15}{2} \\ -8 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{15}{2} \\ -8 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -2 \\ \pi - \frac{59}{4} \end{bmatrix}$$

$$\frac{15}{2}x - 8y + z = 16 + \pi - \frac{59}{4}$$

or $\frac{15}{2}x - 8y + z = \pi + \frac{5}{4}$

2. [4 marks] Use differentials to approximate $f(a+0.4, b-0.3)$ given $f(x,y)=3(\sqrt{x}+\sqrt{y})^2$ and $f(a,b)=c$. Your answer will involve a, b, and c.

$$f = 3(\sqrt{x}+\sqrt{y})^2$$

$$f_x = 6(\sqrt{x}+\sqrt{y})\,\tfrac{1}{2}x^{-1/2} = \frac{3(\sqrt{x}+\sqrt{y})}{\sqrt{x}}$$

$$f_y = 6(\sqrt{x}+\sqrt{y})\,\tfrac{1}{2}y^{-1/2} = \frac{3(\sqrt{x}+\sqrt{y})}{\sqrt{y}}$$

$$df = f_x\,dx + f_y\,dy$$

$$f(x+dx, y+dy) \approx f(x,y) + df$$

$$\approx f(x,y) + \frac{3(\sqrt{x}+\sqrt{y})}{\sqrt{x}}dx + \frac{3(\sqrt{x}+\sqrt{y})}{\sqrt{y}}dy$$

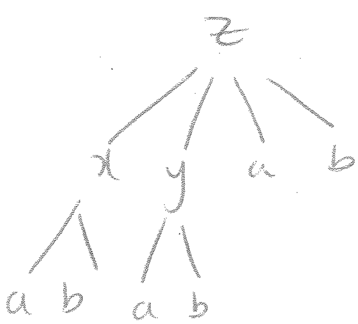
$$\boxed{\text{Sub } x=a,\ dx=0.4,\ y=b,\ dy=-0.3}$$

$$f(a+0.4, b-0.3) \approx \underbrace{\cancel{f(a,b)}}_{c} + \frac{3(\sqrt{a}+\sqrt{b})}{\sqrt{a}}(0.4) + \frac{3(\sqrt{a}+\sqrt{b})}{\sqrt{b}}(-0.3)$$

$$\approx c + 1.2\,\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}} - 0.9\,\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}$$

3. [5 marks] Use the Multivariable Chain Rule to find z_b evaluated at $(a, b) = (2, 3)$ given:

$$z = 3x^3 - 2x^2y^3 + 7y^2 + \frac{3b^2}{a}, \quad x = a^2 - b^2, \quad y = ab.$$



f

$$z_b = f_x\, x_b + f_y\, y_b + f_b$$
$$= (9x^2 - 4xy^3)(-2b) + (-6x^2y^2 + 14y)(a) + \frac{6b}{a}$$

At $(a,b) = (2,3)$: $x = -5$ $y = 6$.

Sub $x = -5$, $y = 6$, $a = 2$, $b = 3$:

$$z_b = (4545)(-6) + (-5316)(2) + 9$$
$$= -37893$$

4. [6 marks] Find the absolute maximum of $z = x^6y^2$ over the region $x^2 + y^2 \leq 2$. Give the absolute maximum value and all points (x, y) where it is achieved.

1) Interior Critical Points

$$\left.\begin{array}{l} z_x = 6x^5y^2 \\ z_y = 2x^6y \end{array}\right\} \text{ both 0 or undefined}$$

$\{(x,y) \text{ in } x^2+y^2 \leq 2 \text{ such that } x=0 \text{ or } y=0\}$

Note: A circle has 0 corners and 1 side.

2) ~~Corners~~

3) Side 1: $x^2+y^2=2$

$y^2 = 2-x^2 \rightarrow z = x^6y^2$

$z = x^6(2-x^2)$

$z = 2x^6 - x^8$

$z' = 12x^5 - 8x^7$

Set $z'=0$: $12x^5 - 8x^7 = 0$

$x^5(12-8x^2) = 0$

$x=0 \qquad 12-8x^2=0$

$x^2 = \frac{12}{8} = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$

$y^2 = 2-x^2$

$y^2 = \frac{1}{2}$

$y = \pm\frac{1}{\sqrt{2}}$

$\longrightarrow$

Points	$z = x^6 y^2$
$x=0$ or $y=0$	0
$\left(\pm\sqrt{\frac{3}{2}}, \pm\frac{1}{\sqrt{2}}\right)$	$\frac{27}{16}$

The absolute maximum is $z = \frac{27}{16}$, achieved at $\left(\pm\sqrt{\frac{3}{2}}, \pm\frac{1}{\sqrt{2}}\right)$.