

Math 250B X01  
Test Two

Time: 50 minutes  
Total: 22 marks

Name: \_\_\_\_\_

1. [4 marks] Set up a **polar double integral** for the following.  
Do not evaluate.

The volume under  $z = x^2y^2$  over the region bounded by  $0 \leq y \leq \sqrt{64 - x^2}$   
and  $-8 \leq x \leq 0$ .

2. [6 marks] Ground temperature (in °C) is given by  $f = 0.2x^2y - 0.1xy^2$ , where  $x$  and  $y$  are measured in km.

a) From  $A = (3, -3)$ , a runner heads towards  $B = (6, 5)$ . What is the runner's initial rate of change of temperature?

b) Starting from  $A = (3, -3)$ , in which direction does  $f$  increase fastest?

c) Starting from  $A = (3, -3)$ , what is the maximum rate of increase of  $f$ ?

3. [6 marks] Evaluate  $\int_0^{16} \int_{\sqrt{y}}^4 \sqrt{1+x^3} \, dx \, dy$ .

4. [6 marks] Use the Lagrange Multiplier method to find the point  $(x, y, z)$  that minimizes  $f = (x - 3)^2 + (y + 6)^2 + (z - 7)^2$ , given  $2x + 2y - z = 12$ .