Math 250B X01
Test Two

Time: 50 Minutes
Total Marks: 25

Name:_____

1. [3 marks] Rewrite the integral using vertical slices instead of horizontal slices. **Do not evaluate the integral.**

$$\int_0^{216} \int_{\sqrt[3]{y}}^{6} \sqrt{1+x^4}\, dx\, dy$$

2. [4 marks] Set up a triple integral for the volume of the solid bounded by $z = x^2$, $y + z = 49$, and $y = 0$. **Do not evaluate the integral.**

3. [6 marks] $T = 0.03x^2y - 0.04xy^2$ gives temperature (in °C) in a small flat town. The variables x and y represent position (in km).

a) A runner travels in a straight line from $(x, y) = (-3, 1)$ to $(x, y) = (8, 7)$. What initial rate of change of temperature does the runner experience?

b) Starting from $(x, y) = (-3, 1)$, in which direction does the temperature increase fastest?

c) Starting from $(x, y) = (-3, 1)$, what is the maximum rate of change of temperature the runner could experience?

4. [6 marks] Solve the following problem using the method of **Lagrange Multipliers**. Given $2x - 2y + 2z = 9$, find the values of x, y, and z that minimize $f = (x-2)^2 + (y+1)^2 + (z+2)^2$.

5. [6 marks] Use a **double integral** in **polar coordinates** to calculate the volume between the cone $z = \sqrt{x^2 + y^2}$ and the hemisphere $z = \sqrt{2 - x^2 - y^2}$.