

1. [3 marks] State whether the surface is a cone, cylinder, paraboloid, plane or sphere:

a) $y = x^2 + z^2$

paraboloid

~~cone~~

b) $z = 7 - \sqrt{x^2 + y^2}$

cone

~~paraboloid~~

c) $y^2 + z^2 = 9$

cylinder

~~paraboloid~~

2. [5 marks] Find all points where the tangent plane to the surface is horizontal.

Give your answer(s) in the form (x, y) .

$z = 2x^4 - 8x^3 + 8x^2 + y^3 - 3y^2$

$z_x = 8x^3 - 24x^2 + 16x$, $z_y = 3y^2 - 6y$

$z_x = 0$

AND

$z_y = 0$

$8x^3 - 24x^2 + 16x = 0$

$8x(x^2 - 3x + 2) = 0$

$8x(x-1)(x-2) = 0$

$x = 0, 1, 2$

$3y^2 - 6y = 0$

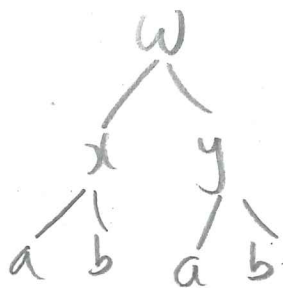
$3y(y-2) = 0$

$y = 0, 2$

$(x, y) = (0, 0), (1, 0), (2, 0), (0, 2), (1, 2), (2, 2)$

3. [6 marks] Use the Multivariable Chain Rule to find w_b at $(a, b) = (0, 0)$ given:

$$w = e^{x^2y} + \ln(xy + 6), \quad x = 1 + \sin(2a + 3b), \quad y = 4 + \cos(\pi - a - 6b).$$



$$\begin{aligned} w_b &= w_x x_b + w_y y_b \\ &= \left[2xy e^{x^2y} + \frac{y}{xy+6} \right] 3 \cos(2a+3b) \\ &\quad + \left[x^2 e^{x^2y} + \frac{x}{xy+6} \right] 6 \sin(\pi - a - 6b) \end{aligned}$$

$$(a, b) = (0, 0) \Rightarrow (x, y) = (1, 3)$$

$$\text{Sub } a=0, b=0, x=1, y=3 :$$

$$\begin{aligned} w_b &= \left[6e^3 + \frac{3}{9} \right] \underbrace{3 \cos 0}_1 + \left[e^3 + \frac{1}{9} \right] \underbrace{6 \sin \pi}_0 \\ &= 18e^3 + 1 \end{aligned}$$

④ marks for differentiation

② marks for evaluation

4. [5 marks] Let $f = \frac{3(\sqrt{x})}{y^7}$

The maximum relative error in x is $\pm 6\%$.

The maximum relative error in y is $\pm 2\%$.

Estimate the maximum relative error in f .

$$f = 3x^{1/2}y^{-7}$$

$$f_x = \frac{3}{2}x^{-1/2}y^{-7}, \quad f_y = -21x^{1/2}y^{-8}$$

$$df = f_x dx + f_y dy$$

$$df = \frac{3}{2}x^{-1/2}y^{-7} dx - 21x^{1/2}y^{-8} dy$$

$$\frac{df}{f} = \frac{\frac{3}{2}x^{-1/2}y^{-7} dx}{3x^{1/2}y^{-7}} - \frac{21x^{1/2}y^{-8} dy}{3x^{1/2}y^{-7}}$$

$$\frac{df}{f} = \frac{1}{2} \frac{dx}{x} - 7 \frac{dy}{y}$$

To maximize $\frac{df}{f}$, use $\frac{dx}{x} = 6\%$ and $\frac{dy}{y} = -2\%$.

$$\begin{aligned} \left(\frac{df}{f}\right)_{\max} &= \frac{1}{2}(6\%) - 7(-2\%) \\ &= 17\% \end{aligned}$$

5. [6 marks] Find the absolute maximum of $z = x^4 y^2$ over the region $x^2 + y^2 \leq 6$.
Give the absolute maximum value and all points (x, y) where it is achieved.



1) Interior Critical Points

$$\left. \begin{aligned} z_x &= 4x^3 y^2 \\ z_y &= 2x^4 y \end{aligned} \right\} \text{ both 0 or undefined}$$

$$\Rightarrow x=0 \text{ or } y=0$$

$\{(x, y) \text{ in } x^2 + y^2 < 6 \text{ such that } x=0 \text{ or } y=0\}$

Note: A circle has 1 side and 0 corners.

2) Critical Points on Side 1: $x^2 + y^2 = 6$

$$y^2 = 6 - x^2 \rightarrow z = x^4 y^2$$

$$z = x^4 (6 - x^2)$$

$$z = 6x^4 - x^6$$

$$z' = 24x^3 - 6x^5$$

$$\text{Set } z' = 0: 24x^3 - 6x^5 = 0$$

$$6x^3(4 - x^2) = 0$$

$\downarrow$
 $x=0$
(already covered)

$$\downarrow$$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$y^2 = 6 - x^2 = 2 \Rightarrow y = \pm \sqrt{2}$$

Point	$z = x^4 y^2$
$x=0$ or $y=0$	0
$(\pm 2, \pm \sqrt{2})$	32 ← Absolute maximum

The absolute maximum is $z=32$,
 achieved at $(x,y) = (\pm 2, \pm \sqrt{2})$.