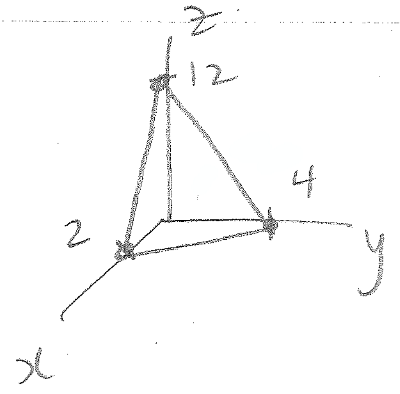
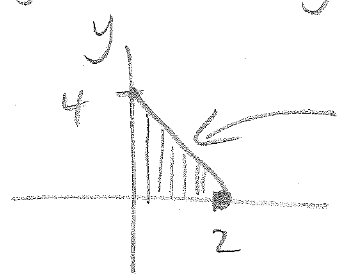
1. [4 marks] Set up a triple integral in rectangular coordinates for the volume under $z = 12 - 6x - 3y$ in the first octant.



$0 \le z \le 12 - 6x - 3y$

Project on xy-plane:



$y = mx + b$

$y = 4 - 2x$

$0 \le y \le 4 - 2x$

$0 \le x \le 2$

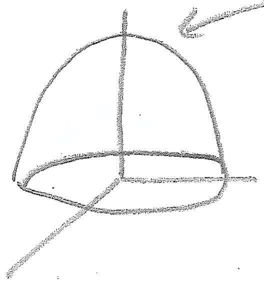
$dV = dz\,dy\,dx$

$V = \iiint dV$

$$V = \int_0^2 \int_0^{4-2x} \int_0^{12-6x-3y} dz\,dy\,dx$$

2. [4 marks] Set up a triple integral in cylindrical coordinates for the volume between the xy-plane and $z = 49 - x^2 - y^2$.

$z=49-r^2$



$0 \le z \le 49-r^2$

Project on xy-plane:

$z = z$

$0 = 49 - r^2$

$r^2 = 49$

$r = \pm 7 \Rightarrow r = 7$

$0 \le r \le 7$

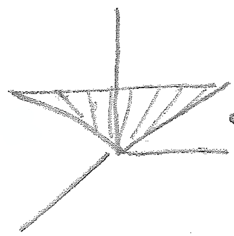
$0 \le \theta \le 2\pi$

$dV = r\,dz\,dr\,d\theta$

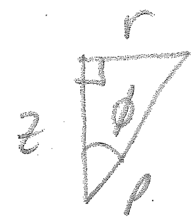
$V = \iiint dV$

$$V = \int_0^{2\pi} \int_0^{7} \int_0^{49-r^2} r\,dz\,dr\,d\theta$$

3. [4 marks] Set up a triple integral in spherical coordinates for the volume between $z = \frac{\sqrt{x^2+y^2}}{2}$ and $z = 1$.



$z = \frac{r}{2}$



ρ: $z = 1$

$\rho\cos\phi = 1$

$\rho = \sec\phi$

$0 \le \rho \le \sec\phi$

ϕ: $z = \frac{r}{2}$

$\rho\cos\phi = \frac{1}{2}\rho\sin\phi$

$2 = \tan\phi$

$\phi = \arctan 2$

$0 \le \phi \le \arctan 2$

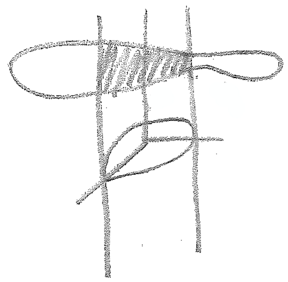
$0 \le \theta \le 2\pi$

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$V = \iiint dV$

$$V = \int_0^{2\pi} \int_0^{\arctan 2} \int_0^{\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

4. [6 marks] Find the surface area of the part of the surface $z = 6 + xy$ that lies inside the cylinder $x^2 + y^2 = 4$. Note: You don't need to graph $z = 6 + xy$ in order to solve the problem.



$z = 6 + xy$

$R: \ x^2 + y^2 = 4$
$r^2 = 4$
$r = \pm 2 \Rightarrow r = 2$
$0 \le r \le 2, \ 0 \le \theta \le 2\pi$

$z_x = y \ , \ z_y = x$

$\sqrt{1 + (z_x)^2 + (z_y)^2} = \sqrt{1 + y^2 + x^2}$

$SA = \iint\limits_R \sqrt{1 + x^2 + y^2} \, dA$

$= \int_0^{2\pi} \int_0^2 \sqrt{1 + r^2} \, r \, dr \, d\theta$

$I = \int \sqrt{1 + r^2} \, r \, dr$
$u = 1 + r^2$
$du = 2r \, dr$
$\frac{du}{2} = r \, dr$
$I = \frac{1}{2} \int \sqrt{u} \, du$
$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$
$= \frac{1}{3} (1 + r^2)^{3/2} + C$

$= \frac{1}{3} (1 + r^2)^{3/2} \Big|_{r=0}^{r=2} \int_0^{2\pi} d\theta$

$= \frac{1}{3} [5^{3/2} - 1] (2\pi)$

$= \frac{2\pi (5^{3/2} - 1)}{3}$

5. [6 marks] Find the work done by the force field $\mathbf{F} = [x + y, x]$ along the straight line segment from $(3, -2)$ to $(6, 7)$.

$C: \quad \vec{x} = \vec{p} + t\vec{d}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + t \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$x = 3 + 3t, \quad y = -2 + 9t \quad (0 \le t \le 1)$

$dx = 3dt, \quad dy = 9dt$

$$W = \int_C (P\,dx + Q\,dy)$$

$$= \int_C [(x+y)dx + x\,dy]$$

$$= \int_0^1 [(1 + 12t)(3dt) + (3+3t)(9dt)]$$

$$= \int_0^1 [3 + 36t + 27 + 27t]\,dt$$

$$= \int_0^1 [30 + 63t]\,dt$$

$$= \left[30t + \frac{63}{2}t^2\right]_0^1$$

$$= \frac{123}{2} \text{ or } 61.5$$