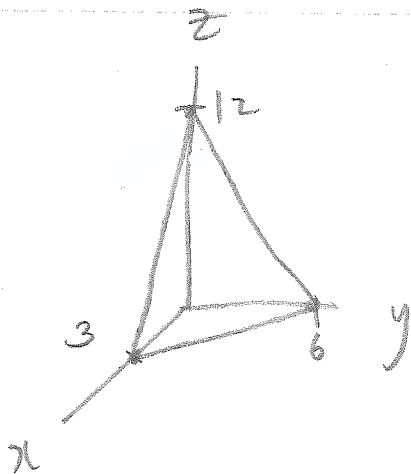
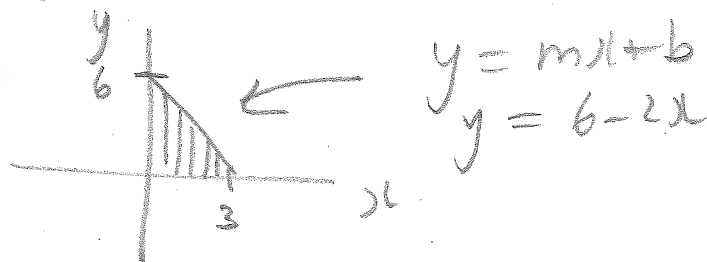


1. [4 marks] Set up a triple integral in rectangular coordinates for the volume under $z = 12 - 4x - 2y$ in the first octant.



$$0 \leq z \leq 12 - 4x - 2y$$

Project on xy -plane :



$$0 \leq y \leq 6 - 2x$$

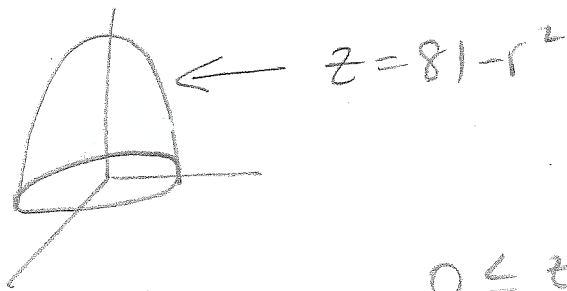
$$0 \leq x \leq 3$$

$$dV = dz dy dx$$

$$V = \iiint dV$$

$$V = \int_0^3 \int_0^{6-2x} \int_0^{12-4x-2y} dz dy dx$$

2. [4 marks] Set up a triple integral in cylindrical coordinates for the volume between the xy -plane and $z = 81 - x^2 - y^2$.



$$0 \leq z \leq 81 - r^2$$

Project on xy -plane:

$$z = z$$

$$0 = 81 - r^2$$

$$r^2 = 81$$

$$r = \pm 9 \Rightarrow r = 9$$

$$0 \leq r \leq 9$$

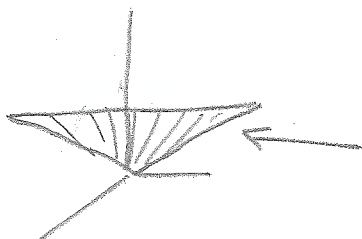
$$0 \leq \theta \leq 2\pi$$

$$dV = r dz dr d\theta$$

$$V = \iiint dV$$

$$V = \int_0^{2\pi} \int_0^9 \int_0^{81-r^2} r dz dr d\theta$$

3. [4 marks] Set up a triple integral in spherical coordinates for the volume between $z = \frac{\sqrt{x^2+y^2}}{3}$ and $z = 1$.



$$z = \frac{5}{3}$$

$$\rho: z = 1$$

$$\rho \cos \phi = 1$$

$$\rho = \sec \phi$$

$$0 \leq \rho \leq \sec \phi$$



$$\phi: z = \frac{5}{3}$$

$$\rho \cos \phi = \frac{1}{3} \rho \sin \phi$$

$$3 = \tan \phi$$

$$\phi = \arctan 3$$

$$0 \leq \phi \leq \arctan 3$$

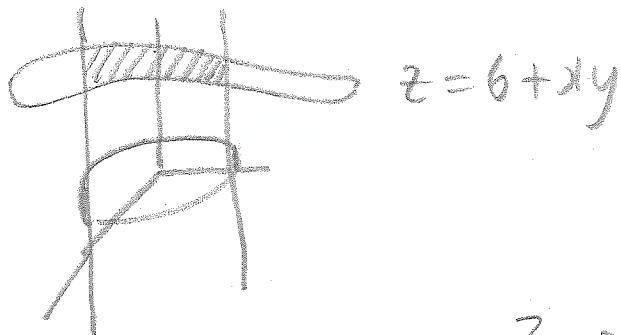
$$0 \leq \theta \leq 2\pi$$

$$dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$V = \iiint dV$$

$$V = \int_0^{2\pi} \int_0^{\arctan 3} \int_0^{\sec \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

4. [6 marks] Find the surface area of the part of the surface $z = 6 + xy$ that lies inside the cylinder $x^2 + y^2 = 9$. Note: You don't need to graph $z = 6 + xy$ in order to solve the problem.



$$R: x^2 + y^2 = 9$$

$$r^2 = 9$$

$$r = \pm 3 \Rightarrow r = 3$$

$$0 \leq r \leq 3, 0 \leq \theta \leq 2\pi$$

$$z_x = y, \quad z_y = x$$

$$\sqrt{1 + (z_x)^2 + (z_y)^2} = \sqrt{1 + y^2 + x^2}$$

$$SA = \iint_R \sqrt{1 + x^2 + y^2} \, dA$$

$$= \int_0^{2\pi} \int_0^3 \sqrt{1 + r^2} \, r \, dr \, d\theta$$

$$I = \int \sqrt{1 + r^2} \, r \, dr$$

$$u = 1 + r^2$$

$$du = 2r \, dr$$

$$\frac{du}{2} = r \, dr$$

$$I = \frac{1}{2} \int \sqrt{u} \, du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} (1 + r^2)^{3/2} + C$$

$$= \frac{1}{3} (1 + r^2)^{3/2} \Big|_{r=0}^{r=3} \int_0^{2\pi} d\theta$$

$$= \frac{1}{3} [10^{3/2} - 1] (2\pi)$$

$$= \frac{2\pi (10^{3/2} - 1)}{3}$$

5. [6 marks] Find the work done by the force field $F = [y, x + y]$ along the straight line segment from $(3, -2)$ to $(6, 7)$.

$$C: \vec{r} = \vec{p} + t\vec{d}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + t \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$x = 3 + 3t, \quad y = -2 + 9t \quad (0 \leq t \leq 1)$$

$$dx = 3dt, \quad dy = 9dt$$

$$W = \int_C (Pdx + Qdy)$$

$$= \int_C [ydx + (x+y)dy]$$

$$= \int_0^1 [(-2+9t)(3dt) + (1+12t)(9dt)]$$

$$= \int_0^1 [-6 + 27t + 9 + 108t] dt$$

$$= \int_0^1 [3 + 135t] dt$$

$$= \left[3t + \frac{135}{2}t^2 \right]_0^1$$

$$= \frac{141}{2} \quad \text{or} \quad 70.5$$