

MATH 250B REVIEW PROBLEMS

1. Find the equation of the plane tangent to $z = y^3 \ln x$ at the point $(e, 2, 8)$.
2. Suppose the temperature T on the disk $x^2 + y^2 \leq 1$ is given by $T = 2x^2 + y^2 - y$. Find the hottest and coldest points on the disk and the temperature at these points.
3. An object's mass m , kinetic energy E and speed v are related by $m = \frac{2E}{v^2}$. Find the maximum relative error in the object's mass given that the relative errors for the kinetic energy and speed are within $\pm 2\%$ and $\pm 3\%$ respectively.
4. Let $f = x^4 - y^4$ where $x = u^2 + v^2$ and $y = u + 2$. Find $\frac{\partial f}{\partial u}$ when $(u, v) = (1, -1)$.
5. Find $\frac{\partial z}{\partial y}$ given $3x^2z - x^2y^2 + 2z^3 + 3yz = 0$
6. The temperature (in $^\circ\text{C}$) on the surface of a metal plate is $T = 20 - 4x^2 - y^2$, where x and y are in cm. In which direction from $(2, -3)$ does the temperature increase most rapidly? What is the rate of increase in this direction?
7. Over a certain region of space the electric potential is $\phi = 5x^2 - 3xy + xyz$, where ϕ is measured in V and position is in m. Find the rate of change of the potential at the point $(3, 4, 5)$ in the direction $\mathbf{i} + \mathbf{j} - \mathbf{k}$.
8. Find the equation of the tangent plane to $x^2 - 2y^2 + z^2 + yz = 2$ at the point $(2, 1, -1)$.
9. Use Lagrange Multipliers to find the points on the surface $z = 4xy + 3$ that are closest to the origin.
10. Find and classify all critical points on the surface $z = 3x^2y + y^3 - 3x^2 - 3y^2$
11. Evaluate $\int_0^2 \int_{3x}^6 e^{-y^2} dy dx$
12. Set up double integrals for the following:
 - a) The area between $y = 4 - x^2$ and $y = 3$
 - b) The volume between $z = 4$ and $z = 2 + x$, over the region bounded by $y = 4 - x^2$ and $y = 3$
13. Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2) dy dx$
14. Find the area bounded by the circles $r = 1$ and $r = 2 \sin \theta$

15. Set up polar double integrals that give the volumes of the following solids:
- The solid bounded by $z = 32 - x^2 - y^2$ and $z = x^2 + y^2$
 - The part of the ball $x^2 + y^2 + z^2 \leq 9$ in the first octant and inside the cylinder $x^2 + y^2 = 3y$
16. Let $\delta = y$. Set up a polar double integral giving the mass of the region bounded by $2 \leq x^2 + y^2 \leq 3, x \geq 0$ and $y \geq \sqrt{3}x$.
17. Set up a triple integral in rectangular coordinates for the volume between $x^2 + y^2 + z^2 = 12$ and $z = x^2 + y^2$.
18. Set up a triple integral in cylindrical coordinates for $\iiint_Q \sqrt{x^2 + y^2} dV$ where Q is the region in the first octant inside $z = \sqrt{25 - x^2 - y^2}$ and outside $x^2 + y^2 = 4x$.
19. Use a triple integral in cylindrical coordinates to find the volume of the solid region inside both the cylinder $x^2 + y^2 = 2y$ and the sphere $x^2 + y^2 + z^2 = 4$.
20. Set up a triple integral in spherical coordinates for the volume of the region bounded by $z = \sqrt{x^2 + y^2}$ and $z = 2$.
21. Set up a triple integral in spherical coordinates for $\iiint_Q \sqrt{x^2 + y^2} dV$ where Q is the region that lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$ and inside the cone $z = \sqrt{9x^2 + 9y^2}$.
22. Evaluate $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$
23. Find the surface area of the portion of the paraboloid $z = 1 + x^2 + y^2$ that lies above the unit disk $x^2 + y^2 \leq 1$.
24. Evaluate $\iint_R y^2 e^{xy} dA$ where R is bounded by $xy = 1, xy = 2, y = 3$ and $y = 4$.
25. Let $\mathbf{F} = x^3 y^2 z \mathbf{i} + xyz \mathbf{j} + xy \mathbf{k}$. Find the divergence and curl of \mathbf{F} .
26. Evaluate $\int_C (x + 2) ds$, where C is given by: $x = t, y = \frac{4}{3}t^{1.5}, z = \frac{t^2}{2}$ for $0 \leq t \leq 2$.
27. Find the work done by $\mathbf{F} = [x, 2y]$ on a particle moving along $y = x^3$ from $(0, 0)$ to $(2, 8)$.
28. Is \mathbf{F} conservative? If so, find a potential function for \mathbf{F} .
- $\mathbf{F} = [y \sin x, \cos x]$
 - $\mathbf{F} = [\frac{1}{y}, -\frac{x}{y^2}, 2z - 1]$

29. If possible, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C begins at $(2, 3)$ and ends at $(7, 6)$.

a) $\mathbf{F} = [xy^2, x^2y]$

b) $\mathbf{F} = [y, -x]$

30. Evaluate $\int_C [y^3 dx + (x^3 + 3xy^2) dy]$ where C is the part of $y = x^2$ from $x = 0$ to $x = 2$ followed by the straight line segment from $(2, 4)$ to $(0, 0)$.

31. Let C be the closed upper semi-circle of radius 9 centred at the origin. Calculate the flux of $\mathbf{F} = [2x^3 + 3y, 2y^3 + 4x]$ across C .

32. Evaluate $\iint_S z \, dS$ where S is given by: $x = 4u \cos v, y = 4u \sin v, z = 3u$ for $0 \leq u \leq 4$ and $0 \leq v \leq 2\pi$.

33. Compute the flux of $\mathbf{F} = [xz, 0, z^2]$ across the first octant of the unit sphere.

34. Find the flux of $\mathbf{F} = [xe^z, ye^z, e^z]$ across the surface bounded by $z = 4 - y, z = 0, y = 0, x = 0$ and $x = 6$.

35. Evaluate $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds$ where $\mathbf{F} = [y^2, -z, -x]$ and C is the boundary of $2x + 2y + z = 6$ in the first octant, oriented counter-clockwise when viewed from above.