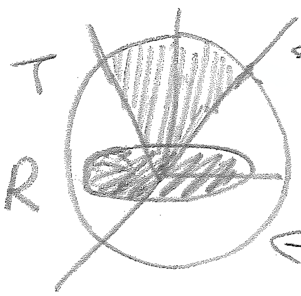
(1) $T: \quad \sqrt{x^2+y^2} \le z \le \sqrt{98-x^2-y^2}$

$$-\sqrt{49-x^2} \le y \le \sqrt{49-x^2}$$

$$-7 \le x \le 7$$



$z = \sqrt{x^2+y^2}$

$\boxed{z = r \text{ in cylindrical}}$

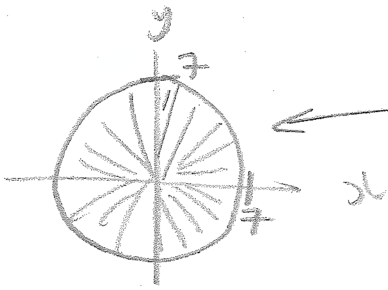
$z = \sqrt{98-x^2-y^2}$ or $\boxed{z = \sqrt{98-r^2} \text{ in cylindrical}}$

$z^2 = 98 - x^2 - y^2$

$x^2+y^2+z^2 = 98$

$\boxed{\text{Conclude} \quad r \le z \le \sqrt{98-r^2}}$

R:



$y = \pm\sqrt{49-x^2}$

$y^2 = 49 - x^2$

$x^2+y^2 = 49$

$r^2 = 49$

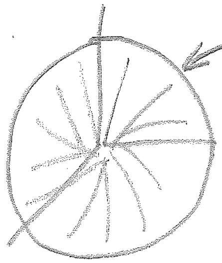
$\cancel{r = \pm 7}$ $r = 7$

$$\boxed{\text{Conclude} \quad \begin{matrix} 0 \le r \le 7 \\ 0 \le \theta \le 2\pi \end{matrix}}$$

$$I = \iiint_T \underbrace{\sqrt{x^2+y^2}}_{r} \, \underbrace{dz\,dy\,dx}_{r\,dz\,dr\,d\theta}$$

$$= \int_0^{2\pi} \int_0^{7} \int_r^{\sqrt{98-r^2}} r^2 \, dz\, dr\, d\theta$$

(2)

T 

$x^2+y^2+z^2=1$

$\rho^2=1$

$\cancel{\rho=\pm 1}$ $\rho=1$

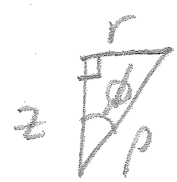
T: $0\le\rho\le 1$

$0\le\phi\le\pi$

$0\le\theta\le 2\pi$

$$I=\iiint_T \underbrace{\sqrt{x^2+y^2}}_{\substack{r\\ =\rho\sin\phi}}\, dV$$

$\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$



$$=\int_0^{2\pi}\int_0^{\pi}\int_0^{1}\rho^3\sin^2\phi\; d\rho\, d\phi\, d\theta$$

$$=\left[\frac{\rho^4}{4}\right]_0^1\int_0^{2\pi}\int_0^{\pi}\underbrace{\sin^2\phi}_{\frac{1}{2}-\frac{\cos 2\phi}{2}}\, d\phi\, d\theta$$

$$=\frac{1}{4}\left[\frac{\phi}{2}-\frac{\sin 2\phi}{4}\right]_0^{\pi}\int_0^{2\pi}d\theta$$

$$=\frac{1}{4}\left(\frac{\pi}{2}\right)\left[\theta\right]_0^{2\pi}$$

$$=\frac{1}{4}\left(\frac{\pi}{2}\right)(2\pi)$$

$$=\frac{\pi^2}{4}$$

③ $\vec{r} = [(1+\cos a)\cos b,\ (1+\cos a)\sin b,\ \sin a]$

$$\vec{r}_a = [-\sin a\cos b,\ -\sin a\sin b,\ \cos a]$$

$$\vec{r}_b = [-(1+\cos a)\sin b,\ (1+\cos a)\cos b,\ 0]$$

$$\vec{r}_a \times \vec{r}_b = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin a\cos b & -\sin a\sin b & \cos a \\ -(1+\cos a)\sin b & (1+\cos a)\cos b & 0 \end{vmatrix}$$

$$= \vec{i}\,[0 - \cos a(1+\cos a)\cos b]$$
$$- \vec{j}\,[0 + \cos a(1+\cos a)\sin b]$$
$$+ \vec{k}\,[-\sin a\cos b(1+\cos a)\cos b - \sin a\sin b(1+\cos a)\sin b]$$

$$= [-\cos a(1+\cos a)\cos b,\ -\cos a(1+\cos a)\sin b,\ -\sin a(1+\cos a)\underbrace{\cancel{(\cos^2 b+\sin^2 b)}}_{1}]$$

$$\|\vec{r}_a \times \vec{r}_b\| = \sqrt{\cos^2 a(1+\cos a)^2\cos^2 b + \cos^2 a(1+\cos a)^2\sin^2 b + \sin^2 a(1+\cos a)^2}$$

$$= \sqrt{\cos^2 a(1+\cos a)^2\underbrace{\cancel{(\cos^2 b+\sin^2 b)}}_{1} + \sin^2 a(1+\cos a)^2}$$

$$= \sqrt{(1+\cos a)^2\underbrace{\cancel{(\cos^2 a+\sin^2 a)}}_{1}}$$

$$= |1+\cos a|$$

$$= 1+\cos a$$

$\rightarrow$

3 Cont'd

$$SA = \iint_R \|\vec{r}_a \times \vec{r}_b\| \, da\,db$$

$$= \int_0^{2\pi}\int_0^{2\pi} (1+\cos a)\, da\, db$$

$$= [a + \sin a]_0^{2\pi} \int_0^{2\pi} db$$

$$= 2\pi \, [b]_0^{2\pi}$$

$$= 2\pi(2\pi)$$

$$= 4\pi^2$$

(4) Let $u = y - 2x$, $1 \le u \le 2$

$v = x + y$, $3 \le v \le 4$

$$\text{Integrand} = (y-2x)e^{3x+3y}$$

$$= (y-2x)e^{3(x+y)}$$

$$= ue^{3v}$$

$$J_{(u,v)\to(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= -3$$

$$J_{(x,y)\to(u,v)} = -\frac{1}{3}$$

$$\left| J_{(x,y)\to(u,v)} \right| = \frac{1}{3}$$

$$\iint_R (y-2x)e^{3(x+y)}\,dA = \int_3^4 \int_1^2 ue^{3v} \underbrace{\frac{1}{3}}_{|J_{(x,y)\to(u,v)}|} du\,dv$$

$$= \frac{1}{3}\left[\frac{u^2}{2}\right]_1^2 \int_3^4 e^{3v}\,dv$$

$\rightarrow$

④ Cont'd

$$= \frac{1}{3}\left(\frac{4}{2}-\frac{1}{2}\right)\left[\frac{e^{3v}}{3}\right]_3^4$$

$$= \frac{1}{3}\left(\frac{3}{2}\right)\left(\frac{e^{12}-e^{9}}{3}\right)$$

$$= \frac{e^{12}-e^{9}}{6}$$

(5) $\vec{F} = [x^2y+z,\ xy^2-xz,\ yz^2+y]$

$$\text{div}\,\vec{F} = \frac{\partial}{\partial x}(x^2y+z) + \frac{\partial}{\partial y}(xy^2-xz) + \frac{\partial}{\partial z}(yz^2+y)$$

$$= 2xy + 2xy + 2yz$$

$$= 4xy + 2yz$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y+z & xy^2-xz & yz^2+y \end{vmatrix}$$

$$= \vec{i}\,[(z^2+1)-(-x)] - \vec{j}\,[0-1] + \vec{k}\,[(y^2-z)-x^2]$$

$$= \vec{i}\,[x+z^2+1] + \vec{j}\,[1] + \vec{k}\,[-x^2+y^2-z]$$

$$= [x+z^2+1,\ 1,\ -x^2+y^2-z]$$