(11) $\int \arctan x \, dx$

$$\boxed{\begin{array}{ll} u = \arctan x & dv = dx \\ du = \dfrac{1}{1+x^2}\,dx & v = x \end{array}}$$

$$\begin{aligned} \int u\,dv &= uv - \int v\,du \\ &= x\arctan x - \int \frac{x}{1+x^2}\,dx \\ &= x\arctan x - \tfrac{1}{2}\ln|1+x^2| + C \end{aligned}$$

(12) $\int \sin^3 \theta \, d\theta$

$= \int \sin^2\theta \sin\theta \, d\theta$

$= \int (1-\cos^2\theta)\sin\theta \, d\theta$

$\boxed{\begin{aligned} \text{Let } u &= \cos\theta \\ du &= -\sin\theta \, d\theta \\ -du &= \sin\theta \, d\theta \end{aligned}}$

$= -\int (1-u^2)\, du$

$= -\left[u - \frac{u^3}{3} \right] + C$

$= -\cos\theta + \frac{\cos^3\theta}{3} + C$

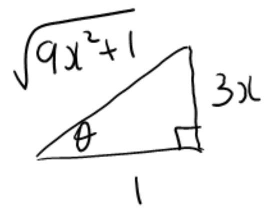
(13) $\int \frac{dx}{\sqrt{9x^2+1}}$

$= \int \frac{dx}{\sqrt{(3x)^2+1}}$

Sub $3x = \tan\theta$

$x = \frac{\tan\theta}{3}$

$dx = \frac{\sec^2\theta}{3} d\theta$



$\frac{\sqrt{9x^2+1}}{1} = \sec\theta$

$\sqrt{9x^2+1} = \sec\theta$

$= \frac{1}{3} \int \frac{\sec^2\theta \, d\theta}{\sec\theta}$

$= \frac{1}{3} \int \sec\theta \, d\theta$

$= \frac{1}{3} \ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{3} \ln\left|\sqrt{9x^2+1} + 3x\right| + C$

(14) Let $\dfrac{2x}{(x+1)(x^2+4)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+4}$

$$\boxed{2x = A(x^2+4) + (Bx+C)(x+1)}$$

Sub $x=-1$: $\quad -2 = 5A \quad \Rightarrow \quad A = \frac{-2}{5}$

x^2 coefficient: $\quad 0 = A+B \quad \Rightarrow \quad B = \frac{2}{5}$

Sub any number, say $x=0$:

$$0 = 4A + C$$
$$C = \frac{8}{5}$$

Integral $= \displaystyle\int \left[\frac{-2}{5} \frac{1}{x+1} + \frac{2}{5} \frac{x}{x^2+4} + \frac{8}{5} \frac{1}{x^2+4} \right] dx$

$= \frac{-2}{5} \ln|x+1| + \frac{2}{5} \cdot \frac{1}{2} \ln|x^2+4| + \frac{8}{5} \cdot \frac{1}{2} \tan^{-1}\frac{x}{2} + C$

$= \frac{-2}{5} \ln|x+1| + \frac{1}{5} \ln|x^2+4| + \frac{4}{5} \tan^{-1}\frac{x}{2} + C$

(15) a) $\lim_{x \to 0} \dfrac{\tan 2x}{\ln(1+x)}$

$\boxed{\text{Form is } \frac{0}{0} \checkmark}$

$\stackrel{(H)}{=} \lim_{x \to 0} \dfrac{2\sec^2 2x}{\left(\frac{1}{1+x}\right)}$

$= \dfrac{2}{1}$

$= 2$

b) Let $L = \lim_{x \to \infty} \left(1 - \frac{1}{x}\right)^{2x}$

$\ln L = \lim_{x \to \infty} \ln\left(1 - \frac{1}{x}\right)^{2x}$

$= \lim_{x \to \infty} 2x \ln\left(1 - \frac{1}{x}\right)$

$= \lim_{x \to \infty} \dfrac{2\ln\left(1 - \frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$

$\boxed{\text{The form is } \frac{0}{0} \checkmark}$

$\stackrel{(H)}{=} \lim_{x \to \infty} \dfrac{2 \frac{1}{\left(1 - \frac{1}{x}\right)} \left(\frac{1}{x^2}\right)}{\left(\frac{-1}{x^2}\right)}$

$= \lim_{x \to \infty} \dfrac{-2}{\left(1 - \frac{1}{x}\right)}$

$= -2$

$L = e^{\ln L} = e^{-2}$

16 $\displaystyle\int_0^\infty x e^{-x}\,dx$

$$= \lim_{b\to\infty} \int_0^b x e^{-x}\,dx$$

	D	I
$+$	x	e^{-x}
$-$	1	$-e^{-x}$
		e^{-x}

$$= \lim_{b\to\infty} \left[-x e^{-x} - e^{-x} \right]_0^b$$

$$= \lim_{b\to\infty} \frac{-b}{e^b} - \frac{1}{e^b} + 1$$

$$= 1$$

$$\boxed{\text{Recall: } \frac{\text{polynomial}}{e^x} \to 0 \quad \text{as} \quad x \to \infty}$$

17 a) $a_1 = -2 \qquad a_2 = 4 \qquad a_3 = -8$

$\lim_{n\to\infty} (-2)^n$ does not exist

b) $a_2 = \frac{6}{9} \qquad a_3 = \frac{9}{13} \qquad a_4 = \frac{12}{17}$

$\lim_{n\to\infty} \frac{3n}{4n+1}$ $\boxed{\text{Form is } \frac{\infty}{\infty} \checkmark}$

$\overset{H}{=} \lim_{n\to\infty} \frac{3}{4}$

$= \frac{3}{4}$

(18) a) Partial Fractions

$$\frac{2}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

$$2 = A(n+2) + B(n+1)$$

sub $n=-1$: $2 = A$

$n=-2$: $2 = -B \Rightarrow B = -2$

$$\sum_{n=5}^{\infty} \frac{2}{(n+1)(n+2)} = \sum_{n=5}^{\infty} \left[\frac{2}{n+1} - \frac{2}{n+2}\right]$$ Telescoping

$$= \left(\frac{2}{6} - \frac{2}{7}\right) + \left(\frac{2}{7} - \frac{2}{8}\right) + \ldots$$

$$= \frac{2}{6} - \lim_{n\to\infty} \frac{2}{n+2}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

b) $$\sum_{n=1}^{\infty} \frac{4(7^{n-1})}{8^n} = \frac{4}{8} + \frac{4(7)}{8^2} + \frac{4(7^2)}{8^3} + \ldots$$

$\boxed{\text{Geometric } a = \frac{4}{8} \quad r = \frac{7}{8}}$

$$= \frac{a}{1-r}$$

$$= \frac{\left(\frac{4}{8}\right)}{\left(\frac{1}{8}\right)}$$

$$= 4$$

19 a) $S_4 = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$

$= \frac{205}{144}$

b) $R_4 \leq \int_4^\infty \frac{1}{x^2}\,dx$

$\leq \lim_{b\to\infty} \int_4^b \frac{1}{x^2}\,dx$

$\leq \lim_{b\to\infty} \left. -\frac{1}{x} \right|_4^b$

$\leq \lim_{b\to\infty} -\frac{1}{b} + \frac{1}{4}$

$\leq \frac{1}{4}$

c) $S_4 \leq \sum_{n=1}^\infty \frac{1}{n^2} \leq S_4 + R_4$

$\frac{205}{144} \leq \sum_{n=1}^\infty \frac{1}{n^2} \leq \frac{241}{144}$

20

$$a_n = \frac{1}{n^2}$$

$$a_{N+1} = \frac{1}{(N+1)^2}$$

Solve $\quad a_{N+1} \leq 0.01$

$$\frac{1}{(N+1)^2} \leq 0.01$$

$$\frac{1}{0.01} \leq (N+1)^2$$

$$100 \leq (N+1)^2$$

$$10 \leq N+1$$

$$9 \leq N$$