FACT 1:

If $f(x)$ is continuous over $[a, \infty)$ then $\int\limits_a^\infty f(x)dx = \lim\limits_{b\to\infty} \int\limits_a^b f(x)dx$.

FACT 2:

If $f(x)$ is continuous over $(-\infty, b]$ then $\int\limits_{-\infty}^b f(x)dx = \lim\limits_{a\to-\infty} \int\limits_a^b f(x)dx$.

FACT 3:

If $f(x)$ is continuous over $(-\infty, \infty)$ then $\int\limits_{-\infty}^\infty f(x)dx = \int\limits_{-\infty}^c f(x)dx + \int\limits_c^\infty f(x)dx$, where c is any real number.

Comments:

An integral converges if its value is a real number. Otherwise it diverges.

The integral in Fact 3 diverges if either integral on the right side diverges.

FACT 4:
Let $f(x)$ be continuous on $[a,b)$ with an asymptote at $x=b$. Then:

$$\int_a^b f(x)dx = \lim_{t \to b^-} \int_a^t f(x)dx.$$

FACT 5:
Let $f(x)$ be continuous on $(a,b]$ with an asymptote at $x=a$. Then:

$$\int_a^b f(x)dx = \lim_{t \to a^+} \int_t^b f(x)dx.$$

FACT 6:
Let $f(x)$ be continuous on $[a,b]$ except at $x=c$ where it has an asymptote. Then:

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

Comment:

The integral in Fact 6 diverges if either integral on the right side diverges.