

1. [3 marks] Evaluate the following limit:

$$\lim_{x \rightarrow -4} \frac{x^2 + 12x + 32}{x^2 - 16}$$

$$= \lim_{x \rightarrow -4} \frac{(x+4)(x+8)}{(x+4)(x-4)}$$

$$= \lim_{x \rightarrow -4} \frac{x+8}{x-4}$$

$$= \frac{4}{-8}$$

$$= -\frac{1}{2}$$

2. [3 marks] An object's displacement (in metres) is given by $s(t) = 0.3t^4 - 8t^2 + 1.9t$, where t is measured in seconds. Find the object's acceleration at $t = 2$ seconds.

$$v(t) = 1.2t^3 - 16t + 1.9$$

$$a(t) = 3.6t^2 - 16$$

$$a(2) = -1.6 \quad \text{m/s}^2$$

3. [5 marks] Use the limit definition to find $f'(x)$ for $f(x) = 7x^2 - 4x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7(x+h)^2 - 4(x+h) - [7x^2 - 4x]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7(x^2 + 2xh + h^2) - 4x - 4h - 7x^2 + 4x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7x^2 + 14xh + 7h^2 - 4h - 7x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{14xh + 7h^2 - 4h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(14x + 7h - 4)}{h}$$

$$= \lim_{h \rightarrow 0} 14x + 7h - 4$$

$$= 14x - 4$$

4. [4 marks] Find $f'(x)$ and simplify fully:

$$f(x) = \frac{3-4x}{7x+2} \quad \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(7x+2)(-4) - (3-4x)(7)}{(7x+2)^2}$$

$$= \frac{-28x - 8 - 7(3-4x)}{(7x+2)^2}$$

$$= \frac{-28x - 8 - 21 + 28x}{(7x+2)^2}$$

$$= \frac{-29}{(7x+2)^2}$$

5. [6 marks] Find y' :

a) $y = \underbrace{2x}_{u}(\underbrace{3x+1}_{v})^7$

$$y' = uv' + vu'$$

$$\begin{aligned} y' &= 2x[7(3x+1)^6(3)] + (3x+1)^7(2) \\ &= 42x(3x+1)^6 + 2(3x+1)^7 \\ \text{or } &2(3x+1)^6(24x+1) \end{aligned}$$

b) $y = 8\sqrt{7x-4}$

$$\begin{aligned} y &= 8(7x-4)^{1/2} \\ y' &= 8 \cdot \frac{1}{2}(7x-4)^{-1/2}(7) \\ &= 28(7x-4)^{-1/2} \end{aligned}$$

c) $y = \frac{-3}{(x^2-6)^4}$

$$\begin{aligned} y &= -3(x^2-6)^{-4} \\ y' &= -3(-4)(x^2-6)^{-5}(2x) \\ &= 24x(x^2-6)^{-5} \end{aligned}$$

6. [4 marks] Find $\frac{dy}{dx}$:

$$5x - 3x^2y^3 = 7y^5 - 12x^2$$

$$5 - [3x^2(3y^2 \frac{dy}{dx}) + y^3(6x)] = 35y^4 \frac{dy}{dx} - 24x$$

$$5 - 9x^2y^2 \frac{dy}{dx} - 6xy^3 = 35y^4 \frac{dy}{dx} - 24x$$

$$-9x^2y^2 \frac{dy}{dx} - 35y^4 \frac{dy}{dx} = 6xy^3 - 5 - 24x$$

$$[-9x^2y^2 - 35y^4] \frac{dy}{dx} = 6xy^3 - 24x - 5$$

$$\frac{dy}{dx} = \frac{6xy^3 - 24x - 5}{-9x^2y^2 - 35y^4}$$

$$\text{or } \frac{dy}{dx} = \frac{24x + 5 - 6xy^3}{9x^2y^2 + 35y^4}$$