16.6 Determinants of 3x3 Matrices

The determinant of A can be written
$\det A$ or $|A|$.

Ex: Find the determinant of $A = \begin{bmatrix} 9 & -6 \\ 2 & 1 \end{bmatrix}$.

$$\det A = 9(1) - (-6)(2) = 21$$

or $$|A| = 9(1) - (-6)(2) = 21$$

FACT
An $n \times n$ matrix A is invertible (has an inverse) exactly when $|A| \neq 0$.

FACT

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

CAUTION

This formula is called <u>cofactor expansion</u>.

Ex: Find the determinant of A, and state whether or not A is invertible.

a) $A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$

$$|A| = 1\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 0 & 7 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 0 & 6 \end{vmatrix}$$

$$= 1(-11) - 4(14) + 6(12)$$

$= 5$

A is invertible

b) $A = \begin{bmatrix} -1 & -4 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 8 \end{bmatrix}$

$|A| = -1 \begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} + 6 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$= -1(6) + 4(6) + 6(0)$

$= 18$

A is invertible

c) $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 9 \\ 1 & -2 & -3 \end{bmatrix}$

$|A| = 1 \begin{vmatrix} 5 & 9 \\ -2 & -3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 9 \\ 1 & -3 \end{vmatrix} + 7 \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix}$

$= 1(3) - 4(-15) + 7(-9)$

$= 0$

A is <u>not</u> invertible.