Term: 2022

Name: Solution Set

Instructor: Patricia Wrean

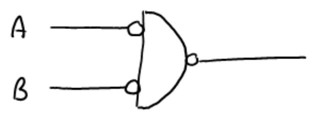
MATH 156-X01
Practice Test 2A

Total = $\overline{30}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app.

GOOD LUCK!

1. (4 points) Consider the following gate diagram.



(a) Write the Boolean expression that corresponds to the diagram. Do not simplify!

$\overline{\bar{A}\,\bar{B}}$

(b) Now simplify your answer to part (a). You do not need to show any work.

$A+B$

(by DeMorgan's)

(−1) each mistake

(−1) using $\wedge$ for "and"
(not Boolean notation)

2. (4 points) Consider the statement, "This soup is hot." let $p=$ "this soup is hot"

Are the following sentences logically equivalent to that statement? Indicate the correct answer by circling "Yes" or "No".

(a) It is not true that this soup is hot. $\bar{p} \not\Leftrightarrow p$ Yes / No

(b) This soup is hot or this soup is hot. $p \vee p \Leftrightarrow p$ idempotent Yes / No

(c) This soup is both hot and spicy or this soup is hot but not spicy. Yes / No

(d) This soup is hot or this soup is not hot but it is spicy. Yes / No

c) $(p \wedge q) \vee (p \wedge \sim q)$

$p \wedge (q \vee \sim q)$ distrib

$p \wedge 1$ complement

p identity

d) $p \vee (\sim p \wedge q)$

$p \vee q$ absorption

3. (3 points) The following statement is true: "If and only if the power fails, then the owlbears will escape." Given that, is it possible for each of the following scenarios to occur? Circle the correct choice.

   Note: You do not need to know what an owlbear is to do this problem.

   (a) The power has not failed and the owlbears have escaped. Yes / No

   (b) The power has not failed and the owlbears have not escaped. Yes / No

   (c) The power has failed and the owlbears have not escaped. Yes / No

4. (3 points) The following statement is true: "If it is raining, then I will bring an umbrella." Given that, answer the following by circling the correct choice.

   (a) I brought an umbrella. Is it raining? Yes / No / Maybe

   (b) I did not bring an umbrella. Is it raining? Yes / No / Maybe

   (c) It is not raining. Did I bring an umbrella? Yes / No / Maybe

5. (4 points) Consider the statement $p \to q$: "If you break a mirror, then you will have seven years of bad luck." Which of the following statements are logically equivalent to $p \to q$? Circle all of the correct answers.

   (a) If you don't break a mirror, you won't have seven years of bad luck.

   (b) If you do not have seven years of bad luck, then you did not break a mirror. $\leftarrow$ contrapositive

   (c) If you have seven years of bad luck, then you broke a mirror.

   (d) Either you did not break a mirror or you had seven years of bad luck or both. $\leftarrow$ "or" form

   $\bar{p} \vee q$

p	q	$\bar{p}$	$p \to q$	$\bar{p} \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

same

6. (5 points) Is $p \to \sim q$ logically equivalent to $\sim(\sim p \leftrightarrow q)$? Use a truth table to justify your answer.

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$\sim p \leftrightarrow q$	$\sim(\sim p \leftrightarrow q)$
0	0	1	1	1	0	1
0	1	1	0	1	1	0
1	0	0	1	1	1	0
1	1	0	0	0	0	1

$\boxed{\text{no}}$

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

7. (5 points) Prove the following using the laws of logic. If you're stuck, try using a truth table for part marks.

$$A + (\overline{C} + 0)(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$$

$A + \overline{C}(\overline{B} + B) = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

$A + \overline{C} \cdot 1 = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ complement

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A} + \overline{A}}$ identity

$A + \overline{C} = \overline{A}\,\overline{C} + \overline{\overline{A}}$ idempotent

$= \overline{A}\,\overline{C} + A$ complement

$= \overline{C} + A$ absorption

8. (2 points) Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression.

$$(\overline{A}B + \overline{A + B})\,\overline{\overline{A}B}$$

$\overline{A+B}\ \overline{\overline{A}B}$ absorption

$\overline{A+B+\overline{A}B}$ De Morgan's

either is fine $\left\{ \begin{array}{ll} \overline{A+B} & \text{absorption} \\ \overline{A}\,\overline{B} & \text{De Morgan's} \end{array} \right.$