

1. [5 marks] Let $A = \{4, 7, 8, 9\}$, $B = \{5, 6, 7, 9\}$ and $C = \{4, 5, 8, 9\}$.

a) Find $(A \cap B) \cup (B \cap C)$

$$A \cap B = \{7, 9\}$$

$$B \cap C = \{5, 9\}$$

$$(A \cap B) \cup (B \cap C) = \{5, 7, 9\}$$

b) Write out all the subsets of A that contain exactly two elements.

$$\{4, 7\}$$

$$\{4, 8\}$$

$$\{4, 9\}$$

$$\{7, 8\}$$

$$\{7, 9\}$$

$$\{8, 9\}$$

2. [2 marks] A six-sided die is rolled twice and we record the total of the two numbers that are rolled. Write out the sample space of this experiment.

The smallest total is $1+1=2$

The largest total is $6+6=12$

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

3. [4 marks] Twelve people apply for a job and eight of them are qualified. How many ways are there to:

$(12-8=4 \text{ unqualified})$

a) Select two qualified applicants and two unqualified applicants for an interview

$$\boxed{C(8,2)} \times \boxed{C(4,2)} = 168$$

Choose 2 qualified from 8 (unordered) Choose 2 unqualified from 4

b) Select four of the applicants for an interview if we select from the entire group of applicants

$$C(12,4) = 495$$

c) Rank the qualified applicants from most qualified to least qualified

$$8! \text{ or } P(8,8) \text{ or } 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

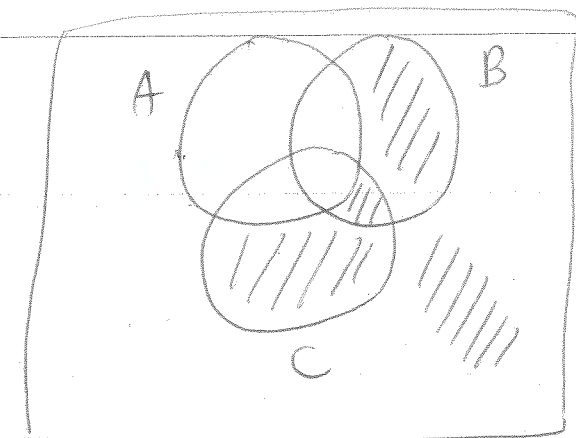
$$= 40,320$$

4. [3 marks] We have three different textbooks (Math, Economics and Statistics) and their solution manuals. We want to arrange these six objects in a row on a bookshelf so that each solution manual is beside the appropriate textbook, either to the left or right. How many different ways are there to do this?

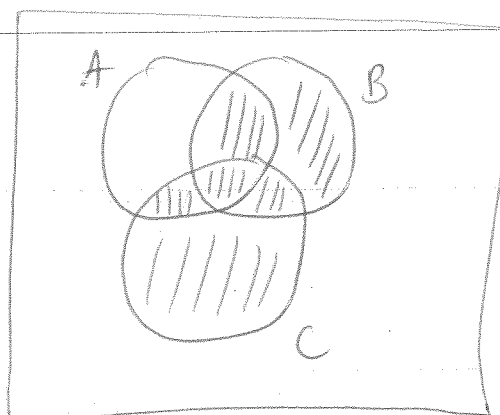
$$\boxed{3!} \times \boxed{2} \times \boxed{2} \times \boxed{2} = 48$$

Arrange the 3 subjects order of Math text and solutions order of Economics text and solutions order of Statistics text and solutions

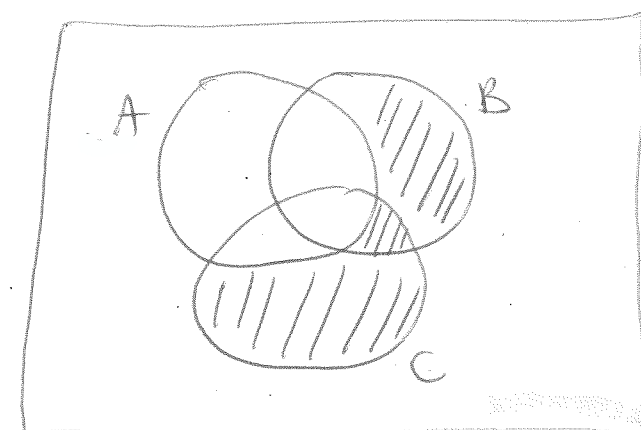
5. [3 marks] Draw a Venn diagram and shade in $A' \cap (B \cup C)$



A'



$B \cup C$



$A' \cap (B \cup C)$

6. [4 marks] An experiment has three possible outcomes: A, B and C. Write down the probability distribution if the probability of A is 0.15 and C is four times as likely as B.

$$\text{Let } \Pr(B) = x$$

Outcome	Probability
A	0.15
B	x
C	$4x$

$$0.15 + x + 4x = 1$$

$$0.15 + 5x = 1$$

$$5x = 0.85$$

$$x = 0.17$$

Outcome	Probability
A	0.15
B	0.17
C	0.68

7. [4 marks] Draw a Venn diagram for the following situation.

Out of 70 students:

23 like to run

25 like to bike

32 like to swim

7 like to run and bike

8 like to run and swim

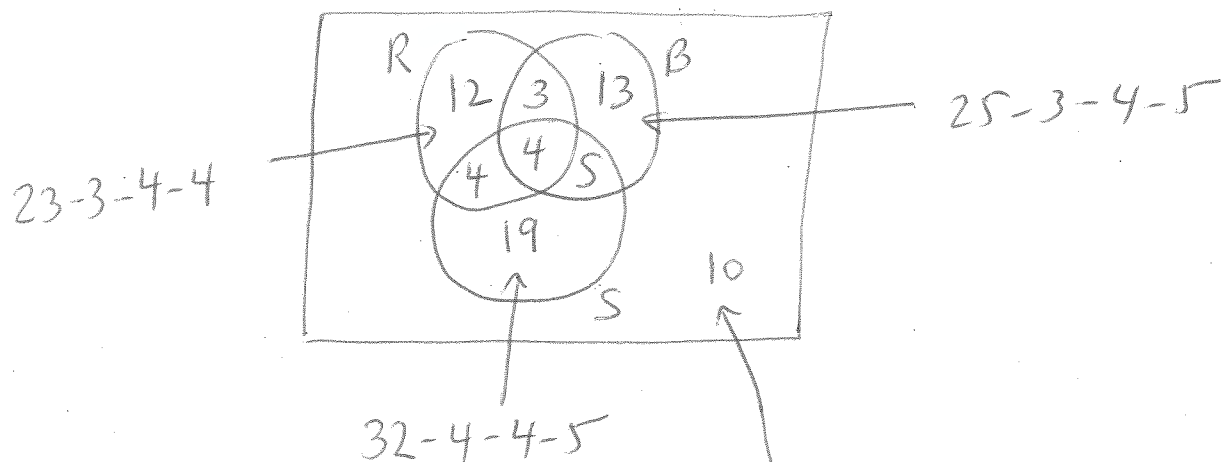
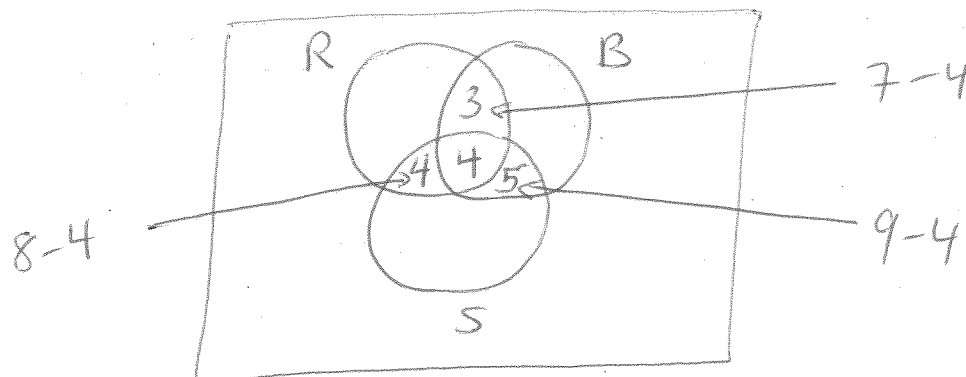
9 like to bike and swim

4 like to run, bike and swim

R: run

B: bike

S: swim



Finally,

70 - 12 - 3 - 13 - 4 - 4 - 5 - 19