FINAL EXAM

Mon April 15
1:30 pm
(three hours)
TEC 174 and 175

Test 2

Fri March 1
2.3 - 2.4, 3.1 - 3.3 (6 Questions)
Bring calculator
Bring music /earplugs
Practice Problems on website

3.5 Subspaces and Basis Cont'd

Subspace of $\mathbb{R}^n$
= span of one or more vectors

e.g. line through origin
plane through origin
all of $\mathbb{R}^3$

Basis for a subspace S

= set of direction vectors for S
that contains the minimum # of vectors

Rowspace of a matrix A = span of the rows of A

Columnspace of a matrix A = span of the columns of A

Example: Let $A = \begin{bmatrix} 2 & 3 & 7 \\ 4 & 7 & 10 \\ 8 & 17 & 8 \end{bmatrix}$. Find a basis for row($A$) consisting of rows of A.

Note: This is different from part a) of the previous example, because that answer was not phrased in terms of rows of A.

$\text{row}(A) = \text{Col}(A^T)$

Find a basis for $\text{Col}(A^T)$.

$$A^T = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 7 & 17 \\ 7 & 10 & 8 \end{bmatrix}$$

$$\frac{R_1}{2} \quad \begin{bmatrix} 1 & 2 & 4 \\ 3 & 7 & 17 \\ 7 & 10 & 8 \end{bmatrix}$$

$$\begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 7R_1 \end{matrix} \quad \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & -4 & -20 \end{bmatrix}$$

$$\begin{matrix} \\ \\ R_3 + 4R_2 \end{matrix} \quad \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{REF}$$

Basis for $\text{Col}(A^T) = \{\text{Columns 1 and 2 of } A^T\}$

$$= \left\{ \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix} \right\}$$

or $\{[2\ 3\ 7], [4\ 7\ 10]\}$

Example: Let $A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 8 & 12 \end{bmatrix}$. Find a basis for null$(A)$.

$\text{null}(A) = \{ \vec{x} \mid A\vec{x} = \vec{0} \}$

Solve $A\vec{x} = \vec{0}$

Each parameter produces a basis vector.

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 0 \\ 2 & 8 & 12 & 0 \end{array}\right]$$

$R_2 - 2R_1$ $$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ RREF

$x_2 = s$

$x_3 = t$

$$x_1 + 4x_2 + 6x_3 = 0 \implies x_1 = -4s - 6t$$

$$\vec{x} = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} t$$

Basis for null(A) $= \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} \right\}$

Example: Find a basis for span$\left(\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\6\end{bmatrix}, \begin{bmatrix}1\\5\\24\end{bmatrix}\right)$.

$$A = \begin{bmatrix}1&1&0\\1&2&6\\1&5&24\end{bmatrix}$$

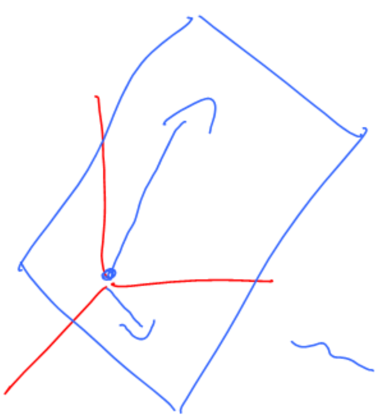
Find a basis for row(A).

$$\begin{matrix}R_2-R_1\\R_3-R_1\end{matrix} \quad \begin{bmatrix}1&1&0\\0&1&6\\0&4&24\end{bmatrix}$$

$$R_3-4R_2 \quad \begin{bmatrix}1&1&0\\0&1&6\\0&0&0\end{bmatrix} \text{ REF}$$

Basis for row$(A) = \{[1 \;\; 1 \;\; 0], [0 \;\; 1 \;\; 6]\}$

or $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\6\end{bmatrix}\right\}$



$\sim$ span$\left(\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\6\end{bmatrix}, \begin{bmatrix}1\\5\\24\end{bmatrix}\right)$