Test 1

Fri Feb 2

1.1-1.4, 2.1-2.2

5 Questions, with parts

Bring calculator

Bring music/earplugs

Practice Problems on website

Example: Solve by Gauss-Jordan Elimination:

$$\begin{aligned} x + 2y + 3z &= 7 \\ 3x + 3y + 3z &= 15 \\ 5x + 7y + 9z &= 29 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 3 & 3 & 3 & 15 \\ 5 & 7 & 9 & 29 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -3 & -6 & -6 \\ 0 & -3 & -6 & -6 \end{array}\right]$$

Current Row − # (Pivot Row)

$\frac{R_2}{-3}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & -6 \end{array}\right]$$

$x \quad y \quad z$

$R_1 - 2R_2$, $R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 0 & -1 & 3 \\ 0 & \textcircled{1} & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF

Circle the leading nonzero entry in each row.

Any column without a circle gets a parameter.

free variable → $z = t$ ← parameter

$x - z = 3 \Rightarrow x = 3 + z \Rightarrow x = 3 + t$

$y + 2z = 2 \Rightarrow y = 2 - 2z \Rightarrow y = 2 - 2t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Example: Solve by Gauss-Jordan Elimination:

$$\begin{aligned} x + y - 6z &= 17 \\ 2x + 2y - 8z &= 22 \\ 3x + 3y - 14z &= 39 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -6 & 17 \\ 2 & 2 & -8 & 22 \\ 3 & 3 & -14 & 39 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -6 & 17 \\ 0 & 0 & 4 & -12 \\ 0 & 0 & 4 & -12 \end{array}\right]$$

$\frac{R_2}{4}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -6 & 17 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 4 & -12 \end{array}\right]$$

$R_1 + 6R_2$

$R_3 - 4R_2$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 1 & 0 & -1 \\ 0 & 0 & \textcircled{1} & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF

$y = k$

$x + y = -1 \implies x = -1 - y \implies x = -1 - k$

$z = -3$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} k$$

Example: Solve by Gauss-Jordan Elimination:

$$\begin{aligned} w + x + 2y + 10z &= 5 \\ x + y + z &= 2 \\ w + 3x + 4y + 12z &= 9 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 12 & 9 \end{array}\right]$$

$R_3 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 & 4 \end{array}\right]$$

$w \quad x \quad y \quad z$

$R_1 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 9 & 3 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 - 2R_2$

RREF

$y = s$ $\quad$ $z = t$

$w + y + 9z = 3 \Rightarrow w = 3 - y - 9z \Rightarrow w = 3 - s - 9t$

$x + y + z = 2 \Rightarrow x = 2 - y - z \Rightarrow x = 2 - s - t$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -9 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Example: Find the intersection of the two lines:

$$\vec{x} = \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \text{ and } \vec{x} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{x} = \vec{x}$$

$$\begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} - t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix}$$

$$s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -6 \end{bmatrix}$$

$$\begin{cases} 2s - t = 0 \\ \cdots \\ \cdots \end{cases}$$

$$\begin{array}{ccc} s & t & \# \\ \left[\begin{array}{c} 2 \\ 1 \\ -1 \end{array}\right. & \left.\begin{array}{c} -1 \\ -1 \\ -1 \end{array}\right. & \left.\begin{array}{c} 0 \\ -2 \\ -6 \end{array}\right] \end{array}$$

To Be Continued...