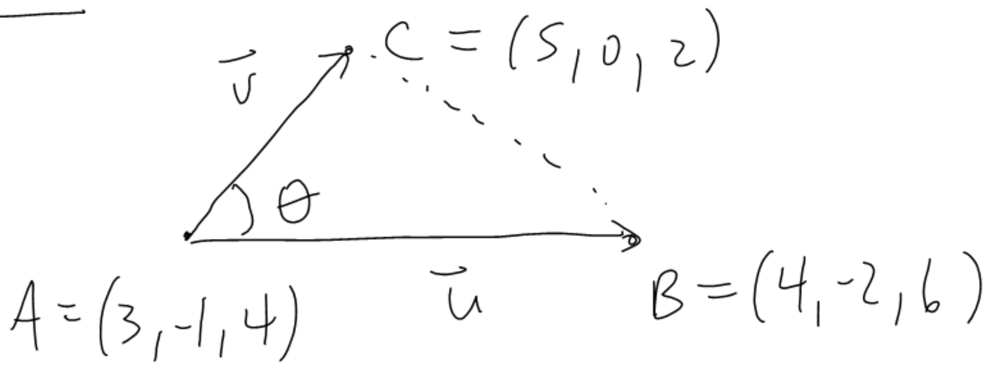
Review 1.2-1.3

Ex: 1.2 #47



Compute $\frac{1}{2}\|\vec{u}\|\,\|\vec{v}-\text{proj}_{\vec{u}}\vec{v}\|$

and $\frac{1}{2}\|\vec{u}\|\,\|\vec{v}\|\sin\theta$

1st part

$$\vec{u}=\overrightarrow{AB}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$\|\vec{u}\|=\sqrt{6}$$

$$\vec{v}=\overrightarrow{AC}=\begin{bmatrix}2\\1\\-2\end{bmatrix}$$

$$\text{proj}_{\vec{u}}\vec{v}=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}$$

$$=\frac{-3}{6}\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$\vec{v}-\text{proj}_{\vec{u}}\vec{v}=\begin{bmatrix}2\\1\\-2\end{bmatrix}+\frac{1}{2}\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$= \begin{bmatrix} 5/2 \\ 1/2 \\ -1 \end{bmatrix} \text{ or } \frac{1}{2}\begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}$$

$$\|\bar{v} - \text{proj}_{\bar{u}}\bar{v}\| = \frac{1}{2}\sqrt{30}$$

$$\frac{1}{2}\|\bar{u}\| \, \|\bar{v} - \text{proj}_{\bar{u}}\bar{v}\| = \frac{1}{2}\sqrt{6}\,\frac{1}{2}\sqrt{30}$$

$$= \frac{\sqrt{180}}{4} \checkmark$$

$$= \frac{6\sqrt{5}}{4} \checkmark$$

$$= \frac{3\sqrt{5}}{2} \checkmark$$

<u>2^{nd} part</u>

$$\bar{u} \cdot \bar{v} = \|\bar{u}\| \, \|\bar{v}\| \cos\theta$$

$$-3 = \sqrt{6}\,(3)\cos\theta$$

$$\frac{-1}{\sqrt{6}} = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{1}{6} + \sin^2\theta = 1$$

$$\sin^2\theta = \frac{5}{6}$$

$$\sin\theta = \pm\sqrt{\frac{5}{6}}$$

$$\boxed{\sin\theta \geq 0 \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ}$$

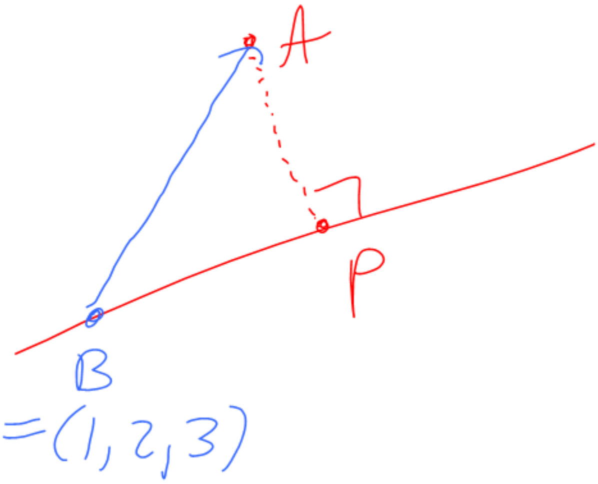
$$\sin\theta = \sqrt{\frac{5}{6}}$$

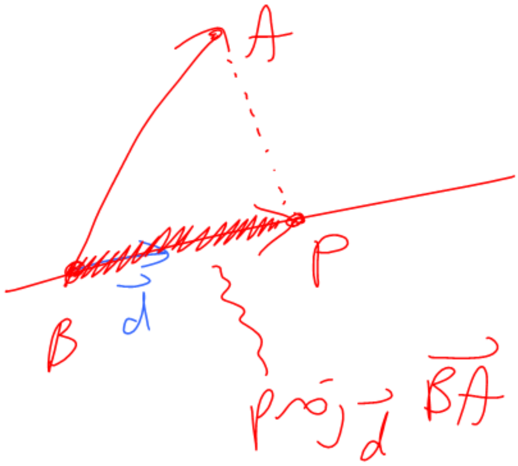
$$\frac{1}{2}\|\vec{u}\|\,\|\vec{v}\|\sin\theta = \frac{1}{2}\sqrt{6}(3)\frac{\sqrt{5}}{\sqrt{6}}$$

$$= \frac{3\sqrt{5}}{2}$$

Ex: Consider $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Find the point P on the line that is closest to $A = (4,2,1)$.





$$\vec{P} = \vec{B} + \text{proj}_{\vec{d}}\,\overrightarrow{BA}$$

$$\vec{BA} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$\text{proj}_{\vec{d}} \vec{BA} = \frac{\vec{BA} \cdot \vec{d}}{\|\vec{d}\|^2} \vec{d}$$

$$= \frac{-8}{14} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{P} = \vec{B} + \text{proj}_{\vec{d}} \vec{BA}$$

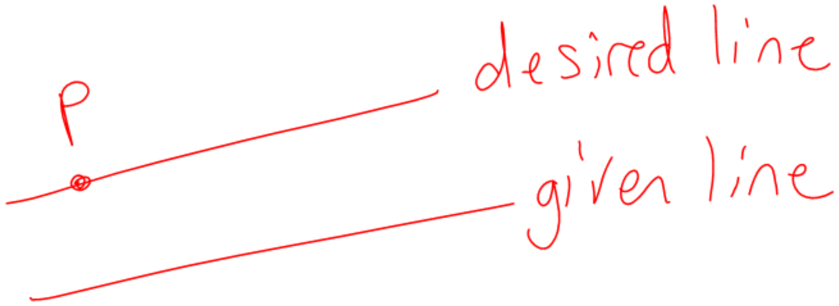
$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{4}{7} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15/7 \\ 2/7 \\ 17/7 \end{bmatrix}$$

$$P = \left(\frac{15}{7}, \frac{2}{7}, \frac{17}{7} \right)$$

Ex: Find the vector form of the line through $P = (-3, 4, 6)$

and parallel to $\begin{cases} x = 7 - t \\ y = 4 + 7t \\ z = 3 + 3t \end{cases}$



$\vec{d} = \begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$ for given line

$\vec{d} = \begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$ for desired line (parallel lines)

given line: $\vec{x} = \vec{p} + t\vec{d}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 6 \end{bmatrix} + t \begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$$