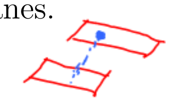
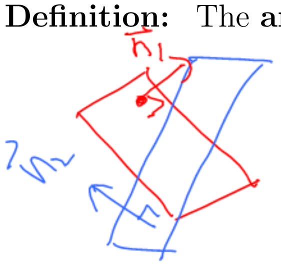
Comment: To find the distance between parallel planes, pick a point on one of the planes. Find the distance between that point and the other plane.



Comment: To find the distance between parallel lines, pick a point on one of the lines. Find the distance between that point and the other line.

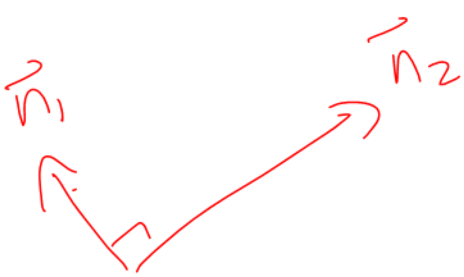
Definition: The **angle between planes** is defined as the angle between their normals.



Definition: **Parallel planes** have parallel normals.
Perpendicular planes have perpendicular normals.

Write an equation that describes
"$\vec{n}_1$ and $\vec{n}_2$ are parallel"

$$\vec{n}_1 = t\,\vec{n}_2$$



Write an equation that describes
"$\vec{n}_1$ is perpendicular to $\vec{n}_2$"

$$\vec{n}_1 \cdot \vec{n}_2 = 0$$

1.4 The Cross Product

The cross product $\vec{u} \times \vec{v}$ is defined for $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$.

Example: Let $\vec{u} = [1, 2, 1]$ and $\vec{v} = [3, -1, 4]$. Calculate $\vec{u} \times \vec{v}$.

$$\begin{matrix} 1 & 2 & 1 & 1 & 2 \\ 3 & -1 & 4 & 3 & -1 \end{matrix}$$

$$\vec{u} \times \vec{v} = [2(4) - 1(-1), 1(3) - 1(4), 1(-1) - 2(3)]$$
$$= [9, -1, -7]$$

Example: Let $\vec{u} = [1, 2, 1]$ and $\vec{v} = [3, -1, 4]$. Calculate:

a) $\vec{v} \times \vec{u}$

$$\begin{matrix} 3 & -1 & 4 & 3 & -1 \\ 1 & 2 & 1 & 1 & 2 \end{matrix}$$

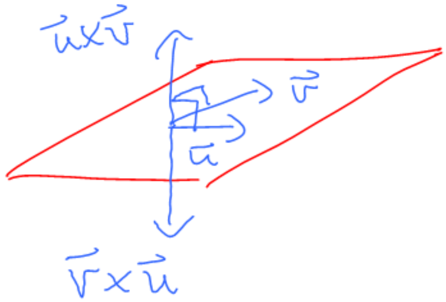
$$\vec{v} \times \vec{u} = [-9, 1, 7]$$

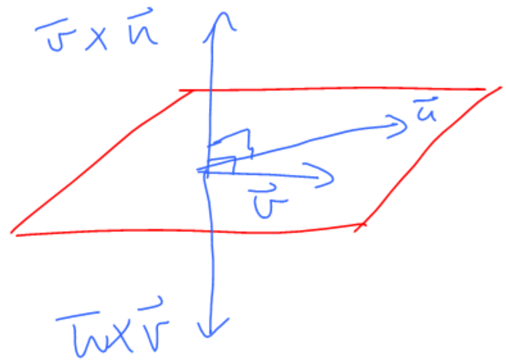
b) $(\vec{u} \times \vec{v}) \cdot \vec{u}$

$$= [9, -1, -7] \cdot [1, 2, 1]$$
$$= 9(1) + (-1)(2) + (-7)(1)$$
$$= 0$$

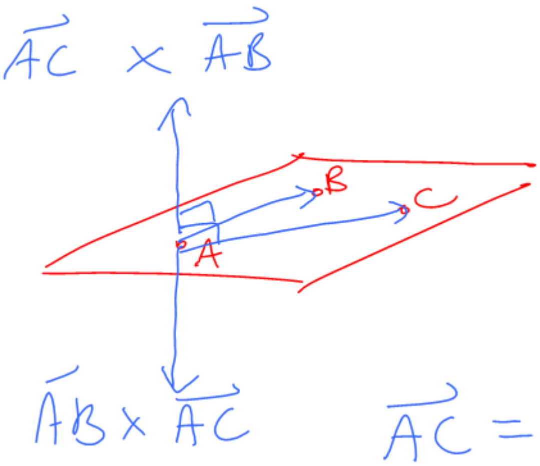
Fact: Let $\vec{u}$ and $\vec{v}$ be in $\mathbb{R}^3$. Then:
$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$ AND
$\vec{u} \times \vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$

Fact: The vector $\vec{u}\times\vec{v}$ is a normal for the plane containing $\vec{u}$ and $\vec{v}$. The direction of $\vec{u}\times\vec{v}$ is determined by the Right Hand Rule.





Example: Find the general form of the plane through $A = (1, 3, 6)$, $B = (2, 1, 4)$ and $C = (1, -1, 5)$.



$$\vec{AC} = [0, -4, -1]$$

$$\vec{AB} = [1, -2, -2]$$

$$\vec{AC}\times\vec{AB} = [6, -1, 4]$$

$$\boxed{\begin{array}{ccccc} 0 & -4 & -1 & 0 & -4 \\ 1 & -2 & -2 & 1 & -2 \end{array}}$$

$$\vec{n} = [6, -1, 4]$$

Normal Form
$$\begin{bmatrix} 6 \\ -1 \\ 4 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 4 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$$

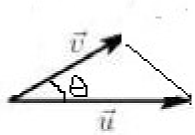
General Form
$$6x - y + 4z = 27$$

or $-6x + y - 4z = -27$

Comment: Recall that $\vec{u}\cdot\vec{v} = ||\vec{u}||\ ||\vec{v}||\cos\theta$ for $\vec{u},\vec{v}$ in $\mathbb{R}^n$.

Fact: If $\vec{u}$ and $\vec{v}$ are in $\mathbb{R}^3$ then $||\vec{u}\times\vec{v}|| = ||\vec{u}||\ ||\vec{v}||\sin\theta$.

Example: Let $\vec{u}$ and $\vec{v}$ be in $\mathbb{R}^3$. Consider the triangle below.
Show that the area of the triangle is $\frac{1}{2}||\vec{u}\times\vec{v}||$



$$\sin\theta = \frac{h}{||\bar{v}||}$$

$$||\bar{v}||\sin\theta = h$$

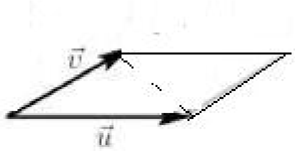
$$h = ||\bar{v}||\sin\theta$$

$$A(\text{triangle}) = \frac{1}{2}bh$$

$$= \frac{1}{2}||\bar{u}||\ ||\bar{v}||\sin\theta$$

$$= \frac{1}{2}||\bar{u}\times\bar{v}||$$

Fact: Let $\vec{u}$ and $\vec{v}$ be in $\mathbb{R}^3$. Consider the parallelogram below, which can be divided into two triangles with equal area. Then:
Area(triangle)= $\frac{1}{2}||\vec{u}\times\vec{v}||$ AND
Area(parallelogram)= $||\vec{u}\times\vec{v}||$



Example: Find the area of the triangle determined by $\vec{u} = [1, 4, 5]$ and $\vec{v} = [2, 3, 6]$.

$$\begin{array}{ccccc} 1 & 4 & 5 & 1 & 4 \\ 2 & 3 & 6 & 2 & 3 \end{array}$$

$$\bar{u} \times \bar{v} = [9, 4, -5]$$

$$\text{Area(triangle)} = \frac{1}{2}\|\bar{u} \times \bar{v}\|$$

$$= \frac{\sqrt{122}}{2}$$