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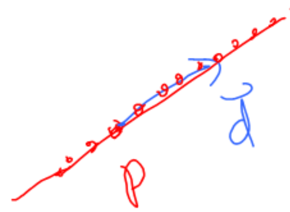
Part 2. Lines in $\mathbb{R}^3$

Example: Consider the line through $P = (2, 1, 12)$ and $Q = (0, -3, 6)$. Describe the line in both vector and parametric form.

vector $\vec{x} = \vec{p} + t\vec{d}$



$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$

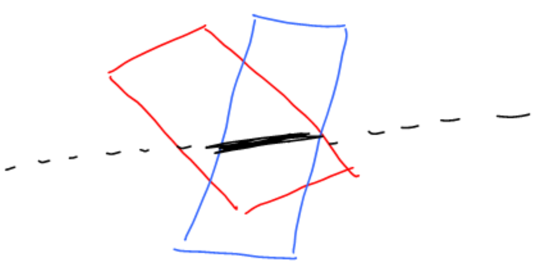
Vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 12 \end{bmatrix} + t \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$

parametric $\begin{cases} x = 2 - 2t \\ y = 1 - 4t \\ z = 12 - 6t \end{cases}$

Definition: A **plane** is an infinite flat surface.

Fact: $ax + by + cz = d$ is the general form for a plane in $\mathbb{R}^3$.

Comment: General form for a line in $\mathbb{R}^3$ is inconvenient so we will omit it. It would consist of two equations, describing the intersection of two planes.

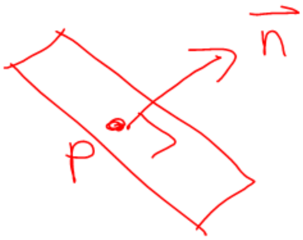


Line : $\begin{cases} 2x + 3y + 5z = 4 \\ x - y + 3z = 7 \end{cases}$

Comment: Similarly we omit normal form for a line in $\mathbb{R}^3$.

Part 3. Planes in $\mathbb{R}^3$

Example: Consider the plane through $P = (1, -1, 3)$ with normal $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Describe the plane in both normal and general form.

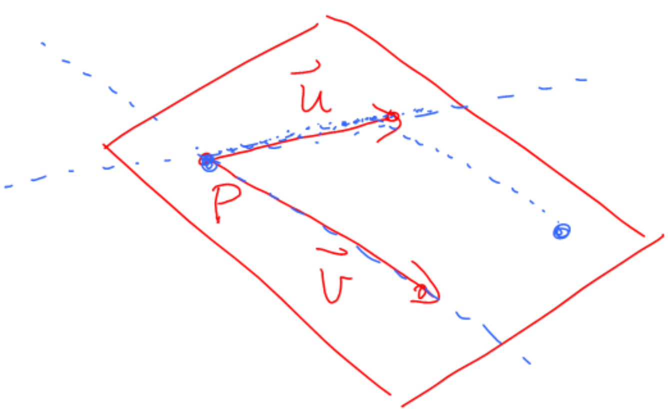


normal $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

normal $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

general $x + y + 2z = 6$

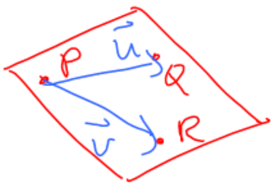
Definition: The **vector form** for a plane in $\mathbb{R}^3$ is $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$ where:
$\vec{u}$ and $\vec{v}$ are nonparallel direction vectors
s and t represent any real numbers



$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

vectorization of any point

Example: Consider the plane through $P = (6, 0, 0)$, $Q = (0, 6, 0)$ and $R = (0, 0, 3)$. Describe the plane in vector and parametric form.



direction vectors $\vec{u} = \overrightarrow{PQ} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$

$\vec{v} = \overrightarrow{PR} = \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$

vector $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} + t \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$

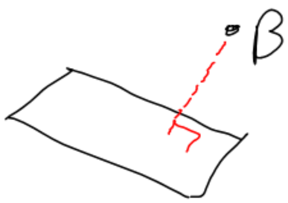
parametric $x = 6 - 6s - 6t$, $y = 6s$, $z = 3t$

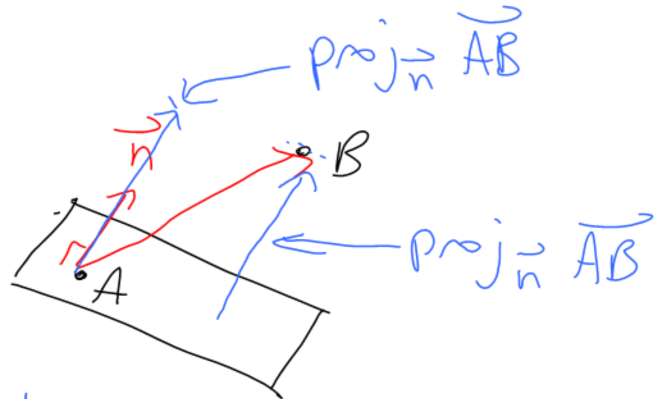
Example: Summarize the twelve descriptions

	Line in $\mathbb{R}^2$	Line in $\mathbb{R}^3$	Plane in $\mathbb{R}^3$
General	$ax + by = c$	×	$ax + by + cz = d$
Normal	$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$	×	$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$
Vector	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$
Parametric	$\begin{cases} x = \\ y = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$

Part 4. Geometry Problems

Example: Find the distance between $B = (1, 3, 3)$ and the plane $\mathcal{P} : x + y + 2z = 7$





$$\text{distance} = \|\text{proj}_{\vec{n}} \overrightarrow{AB}\|$$

$A = (7, 0, 0)$ any point on plane

$$\overrightarrow{AB} = \begin{bmatrix} -6 \\ 3 \\ 3 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

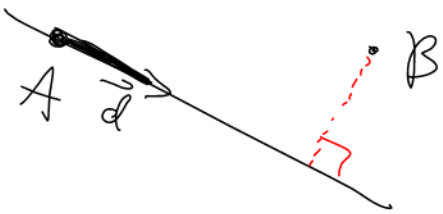
$$\text{proj}_{\vec{n}} \overrightarrow{AB} = \frac{\vec{n} \cdot \overrightarrow{AB}}{\|\vec{n}\|^2} \vec{n}$$

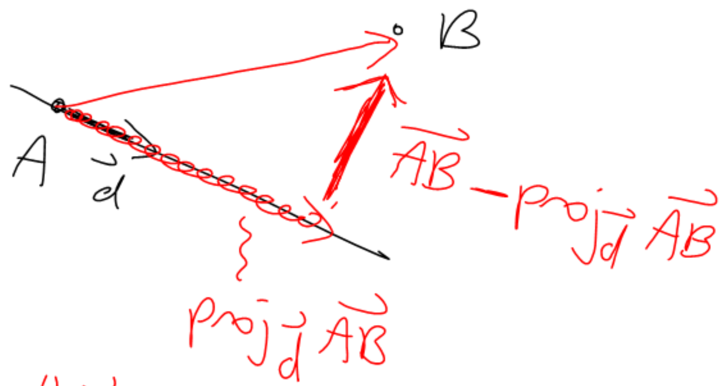
$$= \frac{3}{6} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{distance} = \left\| \frac{3}{6} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\|$$

$$= \frac{3}{6}\sqrt{6} \quad \text{or} \quad \frac{\sqrt{6}}{2}$$

Example: Find the distance between $B = (1,1,0)$ and the line through $A = (0,1,2)$ with $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.





$$\text{distance} = \|\overrightarrow{AB} - \mathrm{proj}_{\vec{d}}\overrightarrow{AB}\|$$

$$\overrightarrow{AB} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\mathrm{proj}_{\vec{d}}\overrightarrow{AB} = \frac{\vec{d}\cdot\overrightarrow{AB}}{\|\vec{d}\|^2}\vec{d} = \frac{-1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\overrightarrow{AB} - \mathrm{proj}_{\vec{d}}\overrightarrow{AB} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 \\ 0 \\ -3/2 \end{bmatrix}$$

$$= \frac{3}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\text{distance} = \left\|\frac{3}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right\|$$

$$= \frac{3}{2}\sqrt{2}$$