NO FORMULA SHEET
FOR MATH 251

Test 1

Fri Feb 2

1.1–1.4, 2.1–2.2

1.3 Lines and Planes

Part 1. Lines in $\mathbb{R}^2$

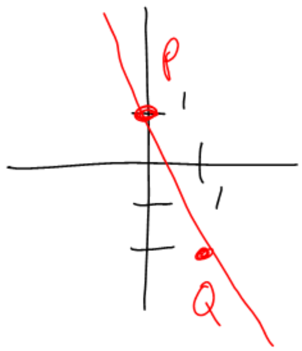
Definition: The **general form** of a line in $\mathbb{R}^2$ is $ax + by = c$

Example: Consider the line $3x + y = 1$. Find two points on the line and graph the line.

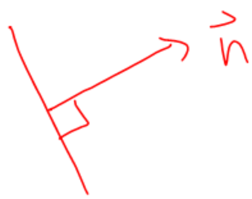
Set $x = 0$: $y = 1$ $\quad P = (0, 1)$

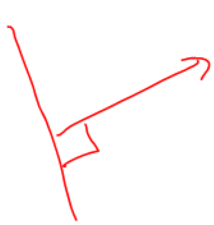
$x = 1$: $3 + y = 1$

$y = -2$ $\quad Q = (1, -2)$



Definition: A **normal vector** is orthogonal to a given line. It is written $\vec{n}$. Its components are the coefficients from the general form.





$\vec{n} = [3, 1]$

$3x + y = 1$

Definition: The **normal form** of a line in $\mathbb{R}^2$ is $\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\vec{p}$ is the vectorization of any point on the line.

Example: Describe the line $3x + y = 1$ in normal form. Show that expanding normal form gives general form.

normal vector $\vec{n} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

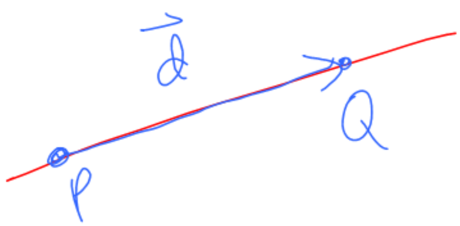
$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad P = (0,1)$ $\quad \vec{p} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

normal form $\quad \vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$

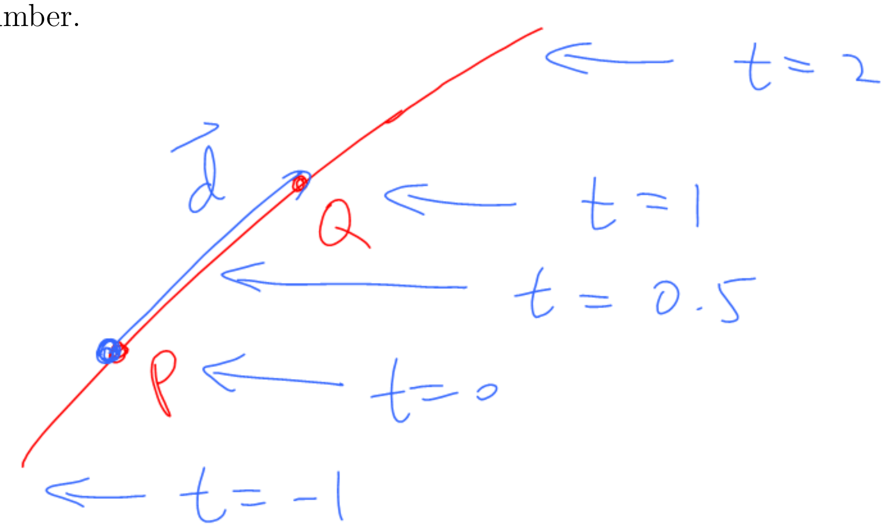
normal form $\quad \begin{bmatrix} 3 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

general form $\quad 3x + y = 1$

Definition: A **direction vector** for a line is $\vec{d} = \overrightarrow{PQ}$, where P and Q are any two points on the line.



Definition: The **vector form** for a line in $\mathbb{R}^2$ is $\vec{x} = \vec{p} + t\vec{d}$, where t represents any real number.



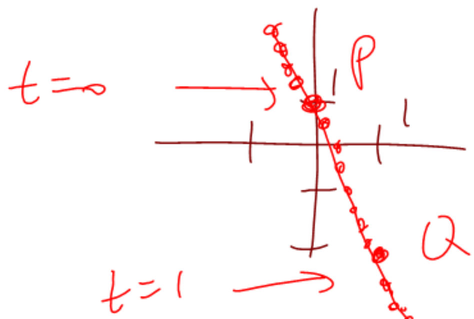
Example: Describe the line $3x + y = 1$ in vector form. Show that as t varies, the line is traced out.

$P = (0,1) \quad Q = (1,-2)$

$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ Think $Q - P$

Vector form $\quad \vec{x} = \vec{p} + t\vec{d}$

Vector form $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \end{bmatrix}$



Definition: The **parametric form** for a line in $\mathbb{R}^2$ is:

$$\begin{cases} x = a + bt \\ y = c + dt \end{cases}$$

Example: Describe the line $3x + y = 1$ in parametric form.

Vector form $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} t \\ -3t \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 1-3t \end{bmatrix}$

Parametric form $\quad \begin{cases} x = t \\ y = 1 - 3t \end{cases}$

Comment: A given line can be described in a specific form in multiple ways, for example $3x + y = 1$ and $6x + 2y = 2$ are general forms for the same line.

Example: Summarize the four forms of a line in $\mathbb{R}^2$

Normal Form $\begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

General Form $3x + y = 1$

Vector Form $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Parametric Form $\begin{cases} x = t \\ y = 1 - 3t \end{cases}$