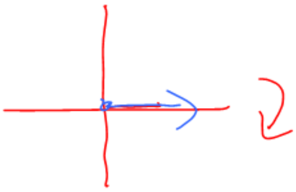
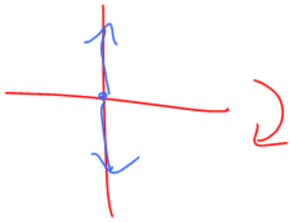
18

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \qquad S\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$





$$S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad S\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \theta = \frac{\pi}{3}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

$$S\left(T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)\right) = [S][T]\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1-\sqrt{3} \\ \sqrt{3}+1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1-\sqrt{3} \\ -\sqrt{3}-1 \end{bmatrix}$$

T_1 first then T_2 :

$$T_2(T_1(\vec{x}))$$

$$A(BC) = (AB)C$$

---

19 $\quad c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 2 & 0 & 6 \\ 1 & 1 & 8 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 5 \end{array}\right]$$

$$c_1 = 3, \quad c_2 = 5$$

$$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) = T\left(3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= T\left(3\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) + T\left(5\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

T is linear

$$= 3\,T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) + 5\,T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

T is linear

$$= 3\begin{bmatrix}5\\6\end{bmatrix} + 5\begin{bmatrix}3\\4\end{bmatrix}$$

$$= \begin{bmatrix}30\\38\end{bmatrix}$$

21 Cramer's Rule

$$y = \frac{|A_2|}{|A|} \qquad A_2 = \begin{bmatrix}3 & 32 & 4\\5 & 39 & -1\\6 & 38 & 1\end{bmatrix}$$

$$|A| = \begin{vmatrix}3 & -1 & 4\\5 & -2 & -1\\6 & 2 & 1\end{vmatrix}$$

$$= 3\begin{vmatrix}-2 & -1\\2 & 1\end{vmatrix} + 1\begin{vmatrix}5 & -1\\6 & 1\end{vmatrix} + 4\begin{vmatrix}5 & -2\\6 & 2\end{vmatrix} \qquad \begin{bmatrix}+ & - & +\end{bmatrix}$$

$$= 3(0) + 1(11) + 4(22)$$

$$= 99$$

$|A_2|$:

$$\begin{array}{ccccc} & & - & - & - \\ 3 & 32 & 4 & 3 & 32 \\ 5 & 39 & -1 & 5 & 39 \\ 6 & 38 & 1 & 6 & 38 \end{array}$$

$$-936 \quad 114 \quad -160 \quad 117 \quad -192 \quad 760$$

$$|A_2| = -297$$

$$y = \frac{|A_2|}{|A|} = -3$$

24 Section 4.3

<u>Recall</u>: If $A\vec{x} = \lambda\vec{x}$ and $\lambda \neq 0$
then $A^{-1}\vec{x} = \lambda^{-1}\vec{x}$.

If $A\vec{x} = \lambda\vec{x}$ then
$$A^n\vec{x} = \lambda^n\vec{x}$$
for integers $n \geq 0$.

Together: If $A\vec{x}=\lambda\vec{x}$ and $\lambda\neq 0$

then $A^{-n}\vec{x}=\lambda^{-n}\vec{x}$

for integers $n\geq 0$.

$$c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$$

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 1 & 3\\ 1 & -1 & 7\end{array}\right]\end{array}$$

$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & 5\\ 0 & 1 & -2\end{array}\right]$$

$$c_1=5,\quad c_2=-2$$

$$\begin{aligned}
A^{-5}\begin{bmatrix}3\\7\end{bmatrix} &= A^{-5}\left(5\begin{bmatrix}1\\1\end{bmatrix}-2\begin{bmatrix}1\\-1\end{bmatrix}\right)\\
&= A^{-5}\left(5\begin{bmatrix}1\\1\end{bmatrix}\right)+A^{-5}\left(-2\begin{bmatrix}1\\-1\end{bmatrix}\right)\\
&= 5A^{-5}\begin{bmatrix}1\\1\end{bmatrix}-2A^{-5}\begin{bmatrix}1\\-1\end{bmatrix}\\
&= 5\underset{32}{\cancel{\left(\tfrac{1}{2}\right)^{-5}}}\begin{bmatrix}1\\1\end{bmatrix}-2\underset{-1}{\cancel{(-1)^{-5}}}\begin{bmatrix}1\\-1\end{bmatrix}\\
&= 160\begin{bmatrix}1\\1\end{bmatrix}+2\begin{bmatrix}1\\-1\end{bmatrix}\\
&= \begin{bmatrix}162\\158\end{bmatrix}
\end{aligned}$$

25

Section 4.4

$$P^{-1}AP = D$$

$$AP = PD$$

$$A = PDP^{-1}$$

$$A^n = PD\cancel{P^{-1}P}DP^{-1} \cdots PDP^{-1}$$

$$A^n = PD^nP^{-1}$$

$$A^7 = PD^7P^{-1}$$

$$D^7 = \begin{bmatrix} 2^7 & 0 \\ 0 & (-3)^7 \end{bmatrix} = \begin{bmatrix} 128 & 0 \\ 0 & -2187 \end{bmatrix}$$

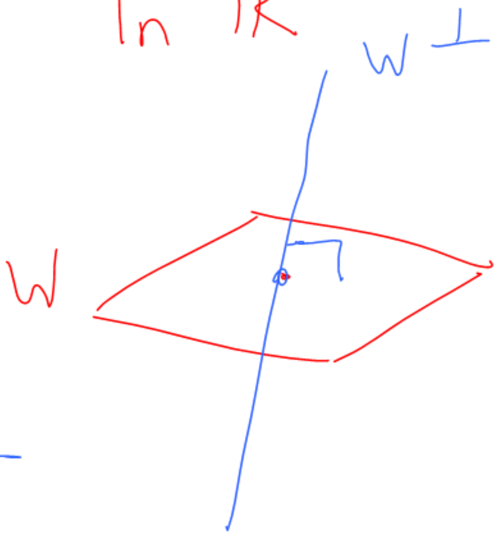
$$P^{-1} = \frac{1}{-8}\begin{bmatrix} 1 & -1 \\ -3 & -5 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -1 & 1 \\ 3 & 5 \end{bmatrix}$$

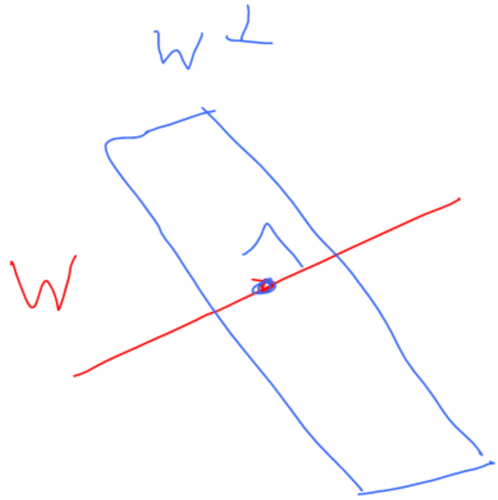
$$A^7 = \frac{1}{8}\begin{bmatrix} -5 & 1 \\ 3 & 1 \end{bmatrix}\left(\begin{bmatrix} 128 & 0 \\ 0 & -2187 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \frac{1}{8}\begin{bmatrix} -5 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -128 & 128 \\ -6561 & -10935 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -5921 & -11575 \\ -6945 & -10551 \end{bmatrix}$$

In $\mathbb{R}^3$





ASIDE

$$\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^n)$$

---

29 Solve $A\vec{x} = \vec{0}$

where A: basis vectors for W in its rows.

Why? $$\begin{bmatrix} \rule{3cm}{0.4pt} \\ \rule{3cm}{0.4pt} \\ \rule{3cm}{0.4pt} \end{bmatrix} \begin{bmatrix} \vec{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 2 & 3 & 10 & 7 & 0 \end{array}\right]$$

$$R_2 - 2R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & 6 & 1 & 0 \end{array}\right]$$

$R_1 - R_2$
$$\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -4 & 2 & 0 \\ 0 & 1 & 6 & 1 & 0 \end{array}\right] \end{array}$$

$y = s \quad z = t$

$$w - 4y + 2z = 0 \implies \begin{array}{l} w = 4s - 2t \\ x = -6s - t \end{array}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} t \qquad (W^\perp)$$

A basis ~~fo~~ for $W^\perp$ $= \left\{ \begin{bmatrix} 4 \\ -6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

ASIDE: In $\mathbb{R}^{12}$

$\dim(W) = 7$

$\implies \dim(W^\perp) = 5$