Omit Review # 10, 20, 22, 36

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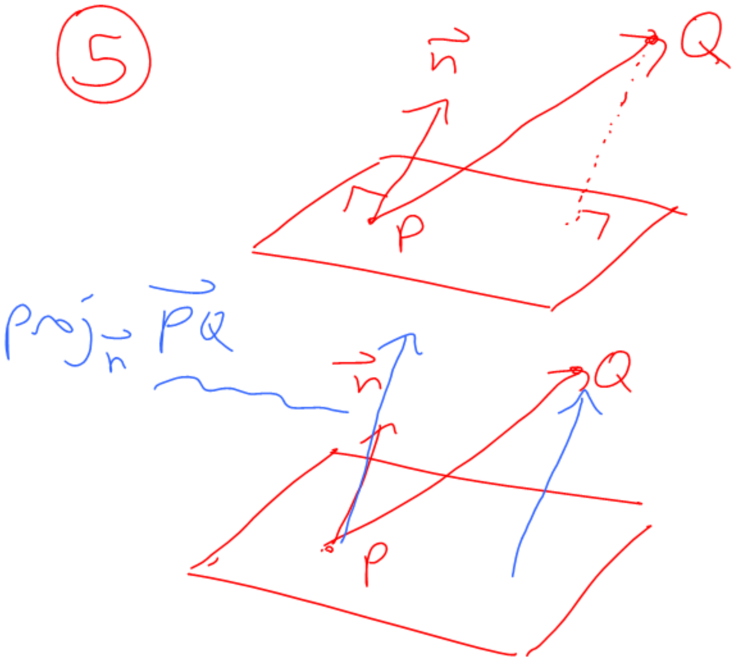

$$\text{Distance} = \| \text{proj}_{\vec{n}} \overrightarrow{PQ} \|$$

Point on plane $P = (0,0,0)$

$$\overrightarrow{PQ} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\text{proj}_{\vec{n}} \overrightarrow{PQ} = \frac{\vec{n} \cdot \overrightarrow{PQ}}{\|\vec{n}\|^2} \vec{n}$$

$$= \frac{13}{35} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\text{Distance} = \left\| \frac{13}{35} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\|$$

$$= \frac{13}{35} \left\| \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\|$$

$$= \frac{13\sqrt{35}}{35}$$

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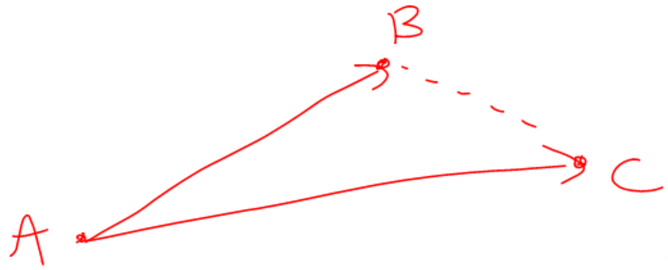

$$\text{area}(\Delta) = \frac{1}{2}\left\|\vec{AB} \times \vec{AC}\right\|$$

Try on your own.

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$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & a \\ 2 & 1 & 3 & b \\ 7 & -19 & 13 & c \end{array}\right]$$

$$\begin{array}{c} \\ R_2-2R_1 \\ R_3-7R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 5 & 1 & a \\ 0 & -9 & 1 & b-2a \\ 0 & -54 & 6 & c-7a \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3-6R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 5 & 1 & a \\ 0 & -9 & 1 & b-2a \\ 0 & 0 & 0 & 5a-6b+c \end{array}\right]_{REF}$$

$$c-7a-6(b-2a)$$
$$= c-7a-6b+12a$$
$$= 5a-6b+c$$

Each zero row gives a condition.

$$\boxed{5a-6b+c=0}$$

(8)

$$\left[\begin{array}{cc|c} 1 & k & 1 \\ k & 1 & 1 \end{array}\right]$$

R_2-kR_1 $\left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1-k^2 & 1-k \end{array}\right]$

$1-k^2 \neq 0$

$\frac{R_2}{1-k^2}$ $\begin{array}{cc|c} x & y & \\ \end{array}$ $\left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1 & \frac{1-k}{1-k^2} \end{array}\right]$

1 solution

$1-k^2=0$

$1=k^2$

$k^2=1$

$k=\pm 1$

$k=1$

$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

∞-many solutions

$k=-1$

$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 2 \end{array}\right]$

no solution

$$\begin{cases} 0, & \text{if } k=-1 \\ 1, & \text{if } k \neq \pm 1 \\ \infty, & \text{if } k=1 \end{cases}$$

(12) $\text{span}\left(\begin{bmatrix} & \end{bmatrix}, \begin{bmatrix} & \end{bmatrix}, \begin{bmatrix} & \end{bmatrix}\right)$

$$= \left\{ c_1\begin{bmatrix} & \end{bmatrix} + c_2\begin{bmatrix} & \end{bmatrix} + c_3\begin{bmatrix} & \end{bmatrix} \right\}$$

$$c_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Get conditions on a, b, c, d.

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 1 & 1 & 0 & b \\ 1 & 0 & 0 & c \\ 1 & 0 & 1 & d \end{array}\right]$$

Continued on next page $\longrightarrow$

ASIDE

Find a basis for $\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$:

$$A = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \end{bmatrix}$$

Find a basis for row(A).

ASIDE

Find a basis for $\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ from among the given vectors:

$$B = \left[\vec{v}_1 \middle| \vec{v}_2 \middle| \vec{v}_3 \right]$$

Find a basis for Col(B)

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12 Cont'd

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & d \\ 0 & 1 & -1 & a-d \\ 0 & 0 & -1 & c-d \\ 0 & 0 & 0 & b-a \end{array}\right]$$

REF

System is solvable if $b - a = 0$

$b = a$

The span is $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, b = a \right\}$

$$= \left\{ \begin{bmatrix} a & a \\ c & d \end{bmatrix} \right\}$$

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$$E_3 E_2 E_1 A = I$$

Elementary matrices act on the left of A.

$$E_3 E_2 E_1 = A^{-1}$$

$$\begin{aligned} A &= (A^{-1})^{-1} \\ &= (E_3 E_2 E_1)^{-1} \quad \text{order reverses!} \\ &= E_1^{-1} E_2^{-1} E_3^{-1} \\ &= \underset{R_2 \leftrightarrow R_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \underset{2R_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}} \underset{R_1 \to R_1 + 3R_3}{\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \end{aligned}$$

$$\boxed{I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

16 a) Nonzero rows of REF/RREF

$$\{ [1 \ 0 \ -1 \ -2], [0 \ 1 \ 2 \ 3] \}$$

b) row(A) = Col(A^T)

Use pivots of RREF of A^T as a guide for which columns of A^T to select.

$$\begin{bmatrix} 1 & & \\ & & 1 \\ & & \end{bmatrix}$$

Use Columns 1 and 3 of A^T.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

c)

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & \end{bmatrix}$$

Use Columns 1 and 2 of A.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \\ 4 \end{bmatrix} \right\}$$

d) $\text{null}(A) = \{\vec{x} \mid A\vec{x} = \vec{0}\}$

$$[A \mid \vec{0}]$$

$$\begin{array}{cccc|c} w & x & y & z & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$y = s \quad z = t$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} t$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$m \times n$ matrix

$\dim(\text{row}(A)) = r = \dim(\text{col}(A))$ ← "rank of A"

$\dim(\text{null}(A)) = n - r$ ← "nullity of A"