Suggested HW for Complex Numbers
Omit #7

Omit # 10, 20, 22, 36
from the Review Problems

Review

Ex: Find the intersection of
$2x+4y+8z=10$ and
$2x+5y+5z=7$. Write your
answer in parametric form.

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 2 & 4 & 8 & 10 \\ 2 & 5 & 5 & 7 \end{array}\right] \end{array}$$

$$\frac{R_1}{2} \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 2 & 5 & 5 & 7 \end{array}\right]$$

$$R_2 - 2R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & -3 \end{array}\right]$$

$R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 10 & 11 \\ 0 & 1 & -3 & -3 \end{array}\right] \quad \text{RREF}$$

$z = t$

$$x + 10z = 11 \implies x = 11 - 10t$$
$$y - 3z = -3 \implies y = -3 + 3t$$

$$\begin{cases} x = 11 - 10t \\ y = -3 + 3t \\ z = t \end{cases}$$

<u>Ex</u>: Find $\operatorname{proj}_W \vec{u}$ where
$W = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} \right)$ and $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix}$.

Caution: Basis for W is <u>not</u> orthogonal.

Gram-Schmidt.

Partial Basis $X = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

$$\vec{v}_2 = \begin{bmatrix}1\\2\\3\\0\end{bmatrix} - \text{proj}_X \begin{bmatrix}1\\2\\3\\0\end{bmatrix}$$

$$= \begin{bmatrix}1\\2\\3\\0\end{bmatrix} - \frac{4}{3}\begin{bmatrix}1\\0\\1\\1\end{bmatrix}$$

$$3\vec{v}_2 = 3\begin{bmatrix}1\\2\\3\\0\end{bmatrix} - 4\begin{bmatrix}1\\0\\1\\1\end{bmatrix}$$

$$= \begin{bmatrix}-1\\6\\5\\-4\end{bmatrix}$$

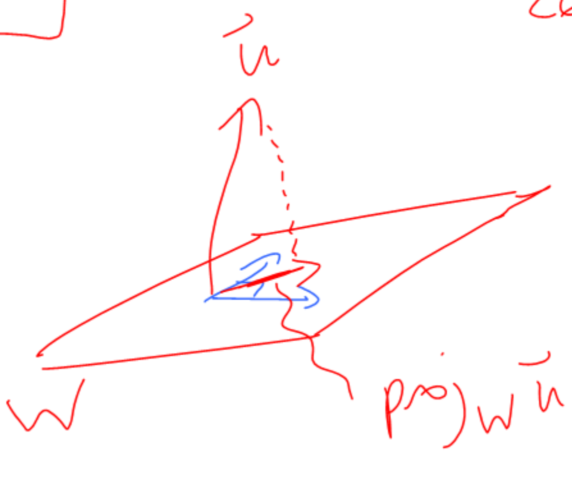
Orthogonal Basis for $W = \left\{ \underbrace{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}}_{\vec{w}_1}, \underbrace{\begin{bmatrix}-1\\6\\5\\-4\end{bmatrix}}_{\vec{w}_2} \right\}$

$$\text{proj}_W \vec{u} = \text{proj}_{\vec{w}_1} \vec{u} + \text{proj}_{\vec{w}_2} \vec{u}$$

$$\boxed{\vec{u} = \begin{bmatrix}1\\2\\4\\1\end{bmatrix}}$$

$$= \frac{6}{3}\begin{bmatrix}1\\0\\1\\1\end{bmatrix} + \frac{27}{78}\begin{bmatrix}-1\\6\\5\\-4\end{bmatrix}$$

$$= \frac{1}{26}\begin{bmatrix}43\\54\\97\\16\end{bmatrix}$$



Ex: Find the eigenvalues of $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 + 1 = 0$$

$$1 - 2\lambda + \lambda^2 + 1 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$\lambda = \frac{2 \pm \cancel{\sqrt{-4}}}{2} \qquad \cancel{\sqrt{4}\sqrt{-1}} \quad 2i$$

$$\lambda = \frac{2 \pm 2i}{2}$$

$$\lambda = 1 \pm i$$

Ex: Set up and solve a system.

Balance $Al + HCl \to AlCl_3 + H_2$

$$w(Al) + x(HCl) \to y(AlCl_3) + z(H_2)$$

Al: $w = y \implies w - y = 0$

H: $x = 2z \implies x - 2z = 0$

Cl: $x = 3y \implies x - 3y = 0$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$R_3 - R_2$ $$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right]$$

$\frac{R_3}{-3}$ $$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right]$$

$R_1 + R_3$

$$\begin{array}{cccc|c} w & x & y & z & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right]$$

RREF

$z = t$

$$w - \frac{2}{3}z = 0 \implies w = \frac{2}{3}t$$

$\dots$ $\quad x = 2t$

$\dots$ $\quad y = \frac{2}{3}t$

Want non-negative integer solutions.

Use $t = 3$.

$$(w, x, y, z) = (2, 6, 2, 3)$$

$$2Al + 6HCl \rightarrow 2AlCl_3 + 3H_2$$

Review Problems

(1) $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

$$-3 = \sqrt{2}\sqrt{29} \cos\theta$$

$$\frac{-3}{\sqrt{2}\sqrt{29}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-3}{(\sqrt{2}\sqrt{29})}\right)$$

$$\approx 113^\circ$$

2 $LS = \|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2$

$= (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$

$+ (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$

$= \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$+ \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$= 2(\vec{u}\cdot\vec{u}) + 2(\vec{v}\cdot\vec{v})$

$= 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$

$= RS$

3, 4 Try on your own.