List of Suggested HW Problems and Full Solutions:

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Problems on D2L

<u>Notation</u>

Vectors are sometimes written in bold font.

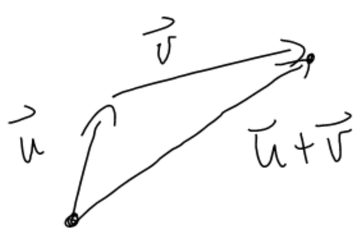
b is a real number

$\vec{b}$ is a vector

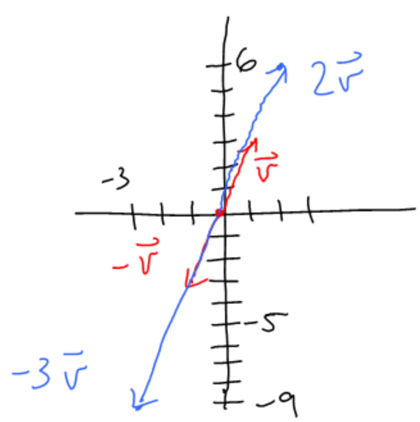
$\mathbf{b}$ is a vector

Example: Let $\vec{u} = [-1, 2]$ and $\vec{v} = [1, 3]$. Find $\vec{u} + \vec{v}$ both algebraically and geometrically.

Example: Graph $\vec{u}, \vec{v}$ and $\vec{u} + \vec{v}$ without a coordinate system.



Example: Let $\vec{v} = [1, 3]$. Graph $2\vec{v}, -\vec{v}$ and $-3\vec{v}$.



$2\vec{v} = [2, 6]$

$-\vec{v} = [-1, -3]$

$-3\vec{v} = [-3, -9]$

Definition: The process of multiplying a vector by a real number is called **scalar multiplication**. It produces a vector that is parallel to the original vector.

Example: Calculate $[2,6]-[3,4]$

$$= [2,6] + [-3,-4]$$

$$= [-1, 2]$$

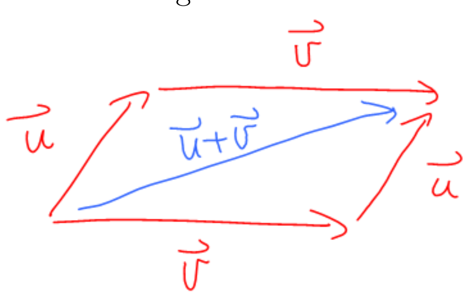
Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Find the vector that runs from the head of $\vec{v}$ to the head of $\vec{u}$.

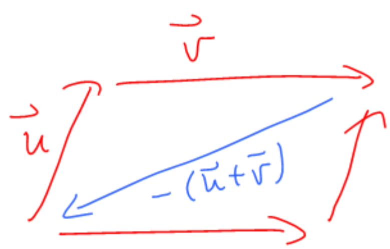
? = backwards along $\vec{v}$ then forwards along $\vec{u}$

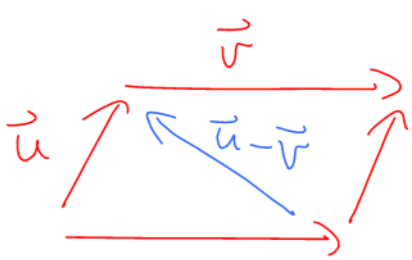
$$= -\vec{v} + \vec{u}$$

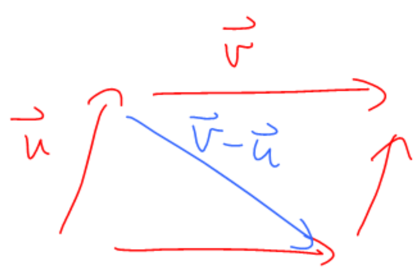
$$= \vec{u} - \vec{v}$$

Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Draw the parallelogram formed by $\vec{u}$ and $\vec{v}$. Label the four diagonals.



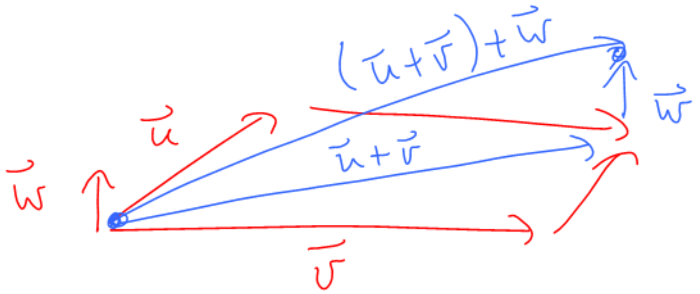


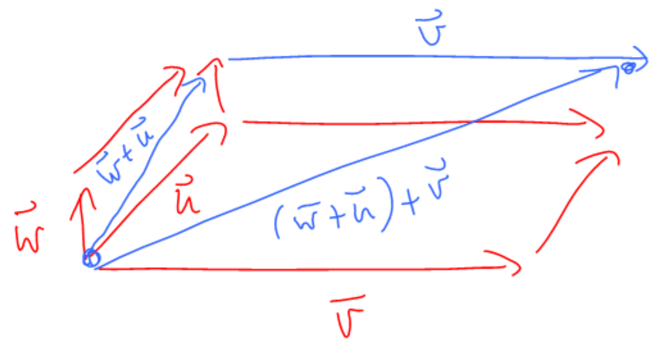




Fact: Order doesn't matter when adding vectors. For any vectors $\vec{u}, \vec{v}$ and $\vec{w}$:
$\vec{u} + \vec{v} = \vec{v} + \vec{u}$
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

Example: Let $\vec{u}, \vec{v}$ and $\vec{w}$ be positioned tail to tail to tail. Show geometrically that
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

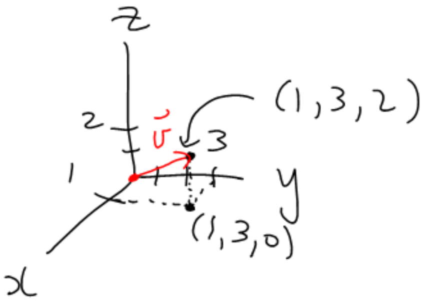


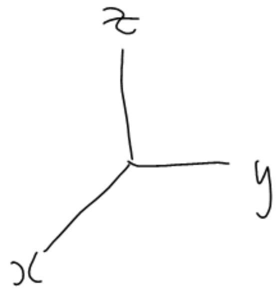


Fact: The above example illustrates that we can write $\vec{u} + \vec{v} + \vec{w}$ without any bracketing.

Definition: Consider the expression: $\vec{v}$ in $\mathbb{R}^n$. This means that $\vec{v}$ has n components, and each component is a real number.

Example: Draw $\vec{v} = [1, 3, 2]$ in $\mathbb{R}^3$.





Definition: The **zero vector** is written $\vec{0}$. Each of its components is zero. The zero vector is useful for algebra.

Example: Write the zero vector in $\mathbb{R}^2$ and $\mathbb{R}^3$.

In $\mathbb{R}^2$: $\vec{0} = [0, 0]$

In $\mathbb{R}^3$: $\vec{0} = [0, 0, 0]$

Example: Let $\vec{u}$ be in $\mathbb{R}^2$. Show (prove) that $\vec{u} + (-\vec{u}) = \vec{0}$.

Let $\vec{u} = [u_1, u_2]$

$$\vec{u} + (-\vec{u}) = [u_1, u_2] + [-u_1, -u_2]$$
$$= [0, 0]$$
$$= \vec{0}$$

Example: Solve for $\vec{x}$ given that $7\vec{x} - \vec{a} = 3(\vec{a} + 4\vec{x})$.

$$7\vec{x} - \vec{a} = 3\vec{a} + 12\vec{x}$$
$$-5\vec{x} - \vec{a} = 3\vec{a}$$
$$-5\vec{x} = 4\vec{a}$$
$$\vec{x} = -\frac{4}{5}\vec{a}$$

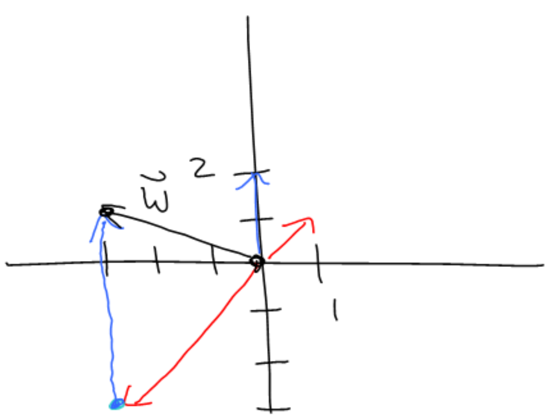
Definition: Consider the statement:
$\vec{w}$ is a **linear combination** of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\2\end{bmatrix}$ with coefficients -3 and 2.
This means that $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$.

Example: Let $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$.

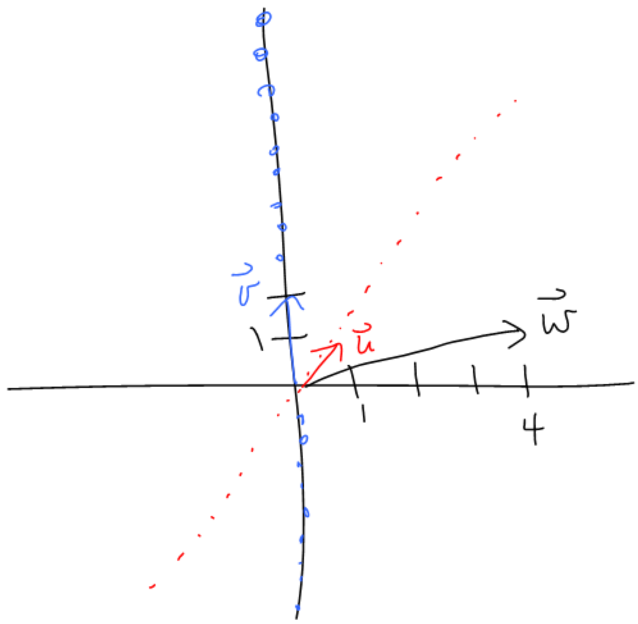
a) Find $\vec{w}$ algebraically.

$$\vec{w} = \begin{bmatrix}-3\\-3\end{bmatrix} + \begin{bmatrix}0\\4\end{bmatrix} = \begin{bmatrix}-3\\1\end{bmatrix}$$

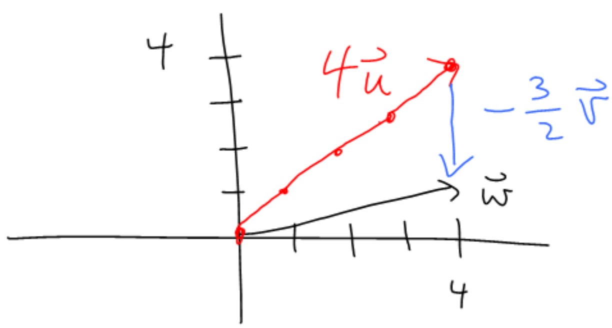
b) Find $\vec{w}$ geometrically.



Example: Write $\vec{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ by graphing.



Think of $\vec{u}$ and $\vec{v}$ as the axes.



$$\boxed{\begin{gathered} \begin{bmatrix} 0 \\ -3 \end{bmatrix} = k \begin{bmatrix} 0 \\ 2 \end{bmatrix} \\ -3 = k(2) \\ -\frac{3}{2} = k \end{gathered}}$$

$$\vec{w} = 4\vec{u} - \frac{3}{2}\vec{v}$$

We'll do this algebraically in Ch. 2

$$\text{Let } \begin{bmatrix} 4 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$\vdots$