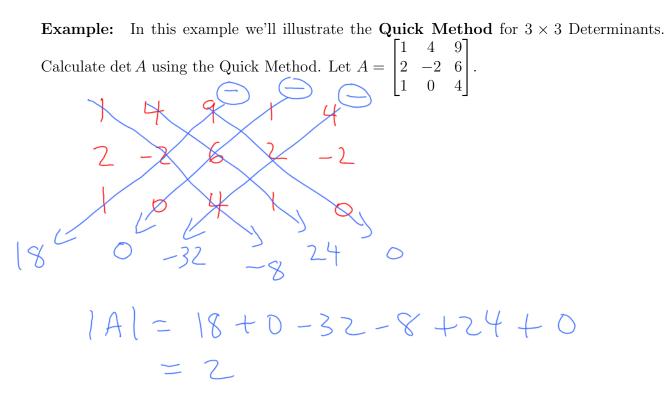
Example: Calculate
$$|A|$$
 for $A = \begin{bmatrix} 1 & 6 & 2 & 3 \\ 0 & 0 & 0 & 4 \\ 2 & 1 & 1 & 6 \\ 2 & 0 & 5 & 7 \end{bmatrix}$



Comment: The Quick Method only applies for 3×3 matrices.

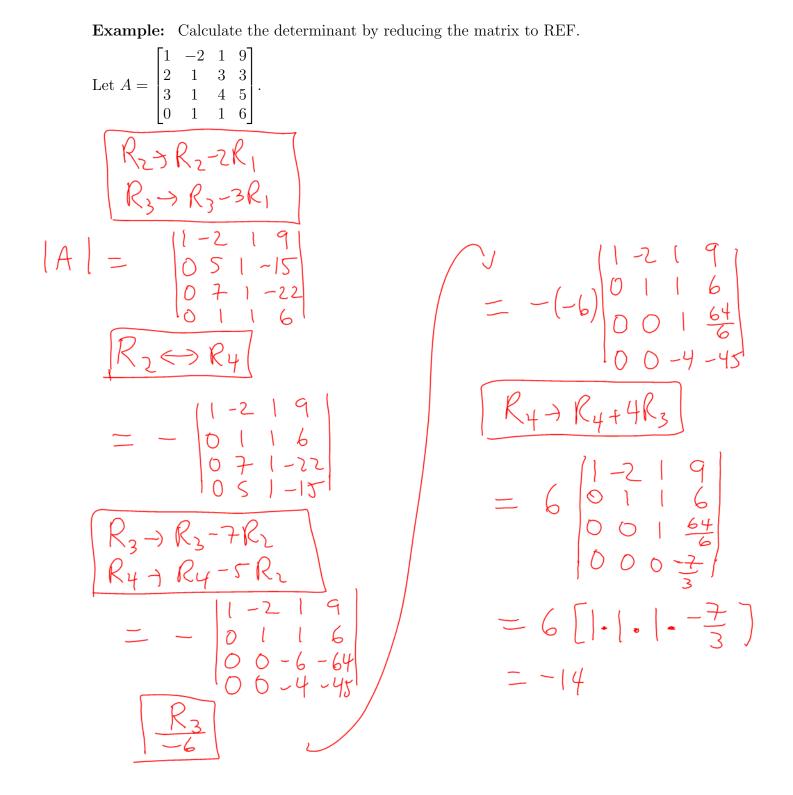
Fact: If A is upper triangular, lower triangular or diagonal then det A is transformed at $\begin{bmatrix} 2 & 9 & 13 \\ 0 & -1 & 1 \\ 0 & 0 & 4 \end{bmatrix} = -8.$ $\begin{bmatrix} 2 & 0 & 0 \\ 9 & 3 & 0 \\ 9 & 9 & 7 \end{bmatrix} = 42$ $\begin{bmatrix} 2 & 9 & 13 \\ 0 & 3 & 0 \\ 9 & 7 \end{bmatrix} = 42$ **Example:** Let's understand why by calculating det $\begin{bmatrix} 2 & 9 & 13 \\ 0 & -1 & 1 \\ 0 & 0 & 4 \end{bmatrix}$. $= 4 \det \begin{bmatrix} 2 & 9 \\ 0 & -1 \end{bmatrix}$ = 4 [2(-1)] = -8

Fact: How Row Operations Change the Determinant:

 $R_i \pm kR_j$ does not change the determinant.

 $R_i \leftrightarrow R_j$ changes the sign of the determinant.

We can factor any row, for example det
$$\begin{bmatrix} 3 & 6 \\ 1 & 5 \end{bmatrix} = 3 \det \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$$
.



Comment: In general det $A \neq \det(\text{REF of } A)$.

Fact: An $n \times n$ matrix A is invertible if and only if det $A \neq 0$.

Fact: Properties of det A: 1) det $A^{-1} = \frac{1}{\det A}$ (if det $A \neq 0$) 2) det $AB = \det A \cdot \det B$ 3) det $kA = k^n \det A$ (where A is $n \times n$) 4) det $A^T = \det A$

Comment: To illustrate Property 3: det $\begin{bmatrix} 7a & 7b \\ 7c & 7d \end{bmatrix} = 7^2 \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. det $\begin{bmatrix} 5a & 5b & 5c \\ 5d & 5e & 5f \\ 5g & 5h & 5i \end{bmatrix} = 5^3 \det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

Comment: Note that $det(A + B) \neq det A + det B$ in general.

Example: Let det $A \neq 0$. Prove Property 1.

$$det A \neq 0$$

$$\Rightarrow A^{-1} exists$$

$$\Rightarrow A^{-1} A = I$$

$$\Rightarrow det (A^{-1} A) = otet I$$

$$\Rightarrow det A^{-1} \cdot det A = 1$$

$$\Rightarrow det A^{-1} \cdot det A = 1$$

Fact: Cramer's Rule Let A be an $n \times n$ matrix. When det $A \neq 0$, the system $A\vec{x} = \vec{b}$ has a unique solution: i-th variable= $\frac{|A_i|}{|A|}$ where $A_i = A$ with the i-th column replaced by \vec{b} .

Example: Solve using Cramer's Rule:

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 0 & 5 \\ 4 & 5 & 1 \end{bmatrix} \qquad \begin{array}{c} 2x + 3y + 2z &= -11 \\ 3x &+ 5z &= 23 \\ 4x + y + z &= 1 \end{array} \qquad A_{2} = \begin{bmatrix} 2 & -11 & 2 \\ 3 & 23 & 5 \\ 4 & 1 & 1 \end{bmatrix}$$

$$z^{nd} = -3 \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} - 5 \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = -3(1) - 5(-10) = 47$$

$$|s^{+} row:$$

$$|A_{2}| = 2 |z_{3} | + || |z_{4} | + || |z_{4} | + || + || |z_{4} | || + || |z_{4} | || + || |z_{4} | || + || || |z_{4} | || || + ||z_{4} | ||| + ||z_{4} | || + ||z_{4} | ||| + ||z_{4} | ||z_{4} | || + ||z_{4} | ||| + ||z_{4} | ||| + ||z_{4} | ||| + ||z_{4} | ||| + ||z_{4} | ||z_{4} | ||| + ||z_{4} | ||z_{4} | ||| + ||z_{4} | ||| + ||z_{4} | ||z_{4} | |||| + ||z_{4} | ||z_{4} | ||| + ||z_{4} | ||z_{4} | |||| + ||z_{4} | ||z_{4} | ||z_{4} | ||||| + ||z_{4} | |z_{4} | |z_{4} | ||z_{4} | ||z_{4} | |z_{4} | |z_{$$