Test Review

(1) Are the vectors linearly dependent? If so, write one vector as a linear combination of the others.

$$\begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\1\end{bmatrix}, \begin{bmatrix}4\\11\\1\end{bmatrix}$$

Let $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

$$\begin{array}{c} \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ 1 & 2 & 4 & 0 \\ -1 & 3 & 11 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2+R_1 \\ R_3-R_1 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 5 & 15 & 0 \\ 0 & -1 & -3 & 0 \end{array}\right]$$

$$\begin{array}{l} \frac{R_2}{5} \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & -3 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1-2R_2 \\ \\ R_3+R_2 \\ \end{array}\left[\begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \textcircled{1} & 0 & -2 & 0 \\ 0 & \textcircled{1} & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF

System has so-many solutions
$\Rightarrow$ Vectors are linearly dependent.

$c_3 = t$

$c_1 - 2c_3 = 0 \Rightarrow c_1 = 2t$

$c_2 + 3c_3 = 0 \Rightarrow c_2 = -3t$

Say $t=1$: $\quad c_1 = 2 \quad c_2 = -3 \quad c_3 = 1$

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$

$$2\vec{v}_1 - 3\vec{v}_2 + \vec{v}_3 = \vec{0}$$

$$\vec{v}_3 = -2\vec{v}_1 + 3\vec{v}_2$$

(2) Set up a system but <u>do not solve</u>.

Find $y = ax^3 + bx^2 + cx + d$ that passes through $(-1,16)$, $(0,7)$, $(1,2)$ and $(10, 2117)$.

a, b, c, d are the variables

$(-1,16) \rightarrow \quad y = ax^3 + bx^2 + cx + d$

$$\begin{cases} 16 = -a + b - c + d \\ 7 = d \\ 2 = a + b + c + d \\ 2117 = 1000a + 100b + 10c + d \end{cases}$$

or

$$\begin{array}{cccc} a & b & c & d \\ \left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 16 \\ & & \ddots & & \end{array}\right. \end{array}$$

(3) Solve using A^{-1}

$$\begin{cases} 2x - 3y = a \\ -3x + 5y = b \end{cases}$$

$$A = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$|A| = 1$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} 5a+3b \\ 3a+2b \end{bmatrix}$$

(4) Write A^{-1} and A as a product of elementary matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$R_2 - 3R_1$ $\quad \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ $\quad E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $(R_2 + 3R_1)$

$\frac{R_2}{-2}$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ $(-2R_2)$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $E_3^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $(R_1 + 2R_2)$

$$\underbrace{E_3 E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_3 E_2 E_1$$

$$A = (A^{-1})^{-1}$$

$$= (E_3 E_2 E_1)^{-1}$$

$$= E_1^{-1} E_2^{-1} E_3^{-1}$$

(5) Solve for X given

$$AXB = (BA)^2$$

Left multiply by A^{-1}

$$\underbrace{A^{-1}A}_{I}XB = A^{-1}(BA)^2$$

$$XB = A^{-1}(BA)^2$$

Right multiply by B^{-1}

$$X\underbrace{BB^{-1}}_{I} = A^{-1}(BA)^2B^{-1}$$

$$X = A^{-1}(BA)^2B^{-1}$$ ✓

or $$X = A^{-1}BABAB^{-1}$$ ✓

(6) Find the general form of

$$\text{span}\left(\begin{bmatrix}1\\3\\6\end{bmatrix}, \begin{bmatrix}2\\6\\4\end{bmatrix}, \begin{bmatrix}3\\9\\5\end{bmatrix}\right)$$

Let $c_1\begin{bmatrix}1\\3\\6\end{bmatrix} + c_2\begin{bmatrix}2\\6\\4\end{bmatrix} + c_3\begin{bmatrix}3\\9\\5\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$

Get conditions on x, y, z

$$\begin{array}{c} C_1\ \ C_2\ \ C_3 \\ \left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 3 & 6 & 9 & y \\ 6 & 4 & 5 & z \end{array}\right] \end{array}$$

$$\begin{array}{l} R_2-3R_1 \\ R_3-6R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 0 & 0 & 0 & y-3x \\ 0 & -8 & -13 & z-6x \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 0 & -8 & -13 & z-6x \\ 0 & 0 & 0 & y-3x \end{array}\right] \quad R+F$$

Solvable system $\Rightarrow$ $y-3x=0$

$y=3x$

$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ such that } y=3x \right\}$$

$$= \left\{ \begin{bmatrix} x \\ 3x \\ z \end{bmatrix} \right\}$$