Test Average = 70% We'll go over #5. (5) How many solutions? $\begin{bmatrix} k \\ k \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$ $R_2 - kR_1 \begin{bmatrix} 0 & 81 - k^2 & 9 - k \end{bmatrix}$ 81-h2 =0 8|-h~= < (9--k)(9+k) = 0 $\frac{R_2}{81-h^2} \begin{bmatrix} 1 & k & | & 1 \\ 0 & 1 & | & \frac{9-k}{81-h^2} \end{bmatrix}$ $\int solution$ k=9 / k=-9Demany no solutions

Fact: To multiply two matrices:

$$AB = \begin{bmatrix} r_1 \cdot c_1 & r_1 \cdot c_2 & \dots & r_1 \cdot c_n \\ r_2 \cdot c_1 & r_2 \cdot c_2 & \dots & r_2 \cdot c_n \\ \dots & \dots & \dots & \dots \\ r_n \cdot c_1 & r_n \cdot c_2 & \dots & r_n \cdot c_n \end{bmatrix}$$

where r_i is row *i* of matrix *A* and c_j is column *j* of matrix *B*.

Example: Find AB where
$$A = \begin{bmatrix} 1 & 4 \\ -2 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

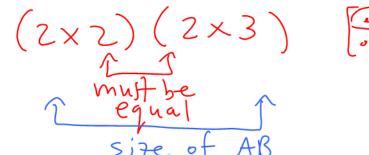
$$\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 6 \end{bmatrix} = 0$$

Example: Let's consider the sizes of A and B in the example above. Which two numbers must be equal to make AB defined? Which two numbers predict the size of AB?





Example: Let A be a 2×3 matrix and let B be a 3×1 matrix. Calculate the sizes of AB and BA.

AB: BA:

$$(2x3)(3x1)$$

= -
 $(3x1)(2x3)$

BA is indefined

AB is ZX1

Fact: $AB \neq BA$ in general.

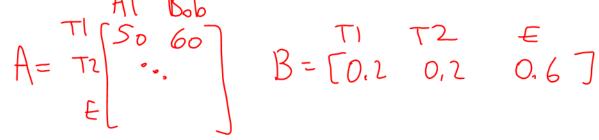
Example: Find *BC* and *CB* where $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. $= \begin{bmatrix} 5 \\ 11 \\ 17 \\ 19 \end{bmatrix}$ CB is undefied CB = **Example:** Expand the following: $\frac{5}{6}$ $\frac{-1}{3}x + \frac{2}{4}y = \begin{bmatrix} 5\\ 6 \end{bmatrix}$ 1x+2y = 53x + 4y = 6



Fact: A system of equations can be written as $A\vec{x} = \vec{b}$ where: A is the coefficient matrix \vec{x} is the vector of variables, written as a column \vec{b} is the vector of constants, written as a column

Example: Consider the data below:			
	Al	Bob	
Test1 Mark	50	60	
Test2 Mark	90	80	
Test2 Mark Exam Mark	75	70	
Test1 Weight Test2 Weight Exam Weight			
0.2		0.2	0.6

Let A be a matrix containing the course marks for the two students. Let B be a matrix containing the weightings of the coursework. Find Al and Bob's final grades using a matrix multiplication.



Need compatible sizes
and compatible categories.

$$AI = \begin{bmatrix} TI & TZ & E \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} TI & S0 & 60 \\ 90 & 80 \\ F & 70 \end{bmatrix}$$

 $= \begin{bmatrix} AI & B_{0}b \\ (73 & 70 \end{bmatrix}$

Definition: The **outer product expansion of** AB is: $AB = A_1B_1 + A_2B_2 + \ldots + A_nB_n$ where A_i is column *i* of *A* and B_j is row *j* of *B*.

Comment: Normal matrix multiplication involves **rows** of the first matrix and **columns** of the second matrix.

The outer product expansion involves **columns** of the first matrix and **rows** of the second matrix.

Example: Find the outer product expansion of *AB* given: $A = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \text{ and } B = \frac{\begin{bmatrix} 1 & 7 \\ 4 & 2 \end{bmatrix}^{-1}}{\begin{bmatrix} 4 & 2 \end{bmatrix}^{-1}}$ $AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ -2 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} \begin{bmatrix} 4 & 2 \end{bmatrix}$ $= \begin{bmatrix} 1 & 7 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ -8 & -4 \end{bmatrix}$ $= \begin{bmatrix} 13 & 13 \\ -8 & -4 \end{bmatrix}$

Example: Confirm the result in the previous example using normal matrix multiplication.

$$AB = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ 4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 13 & 13 \\ -8 & -4 \end{bmatrix}$$

Comment: The outer product expansion will be used further in Section 5.4.

Definition: The expression A^n means multiply A with itself n times. For example: $A^2 = AA$ $A^3 = A^2A$ or $A^3 = AA^2$ or $A^3 = AAA$

Example: Express A^{12} as the cube of a matrix.

$$A^{12} = \left(A^{4}\right)^{3}$$

Example: Compute
$$A^2$$
 for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{pmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Fact: Recall that I is the identity matrix. For any matrix A: AI = A and IA = A

Example: Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$
. Confirm that $AI = A$ and $IA = A$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Example: Simplify B^{2018} given that $B^3 = I$.

$$2 \circ 18 = ?(3) + ?$$

$$\frac{2 \circ 18}{3} \approx 672.7$$

$$2 \circ 18 = 672(3) + ?$$

$$2 \circ 18 = 672(3) + 2$$

$$2 \circ 18 = 3(672) + 2$$

$$B = B$$

$$= B^{3(672)} + 2$$

$$B = B^{3(672)} + 2$$

$$= B^{2}$$

$$= B^{3(672)} + B^{2}$$

$$= B^{2}$$

$$= B^{2}$$

Definition: Let **O** be the zero matrix. For example $\mathbf{O} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ etc.

Example: Find a 2×2 matrix A so that $A^2 = \mathbf{O}$ but $A \neq \mathbf{O}$.

$$\begin{bmatrix} -2 & 2 \\$$