

Test Review

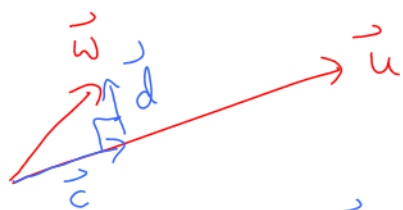
Ex: Let $\vec{u} = [1, -1, 2]$ and $\vec{w} = [3, 0, 4]$.

Find \vec{c} and \vec{d} so that:

\vec{c} is parallel to \vec{u} ,

\vec{d} is perpendicular to \vec{u} , and

$$\vec{w} = \vec{c} + \vec{d}$$



$$\begin{aligned}\vec{c} &= \text{proj}_{\vec{u}} \vec{w} \\ &= \frac{\vec{u} \cdot \vec{w}}{\|\vec{u}\|^2} \vec{u} \\ &= \frac{11}{6} [1, -1, 2]\end{aligned}$$

$$\vec{w} = \vec{c} + \vec{d}$$

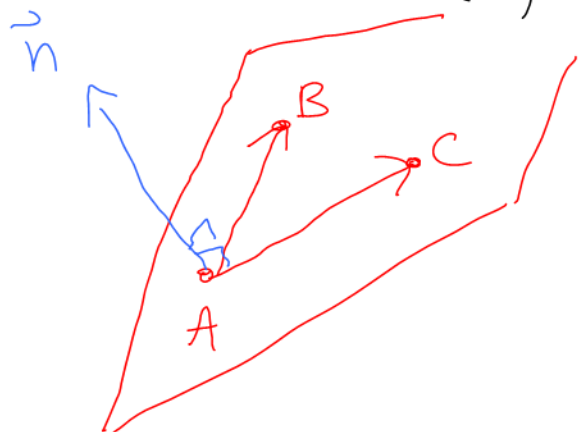
$$\vec{d} = \vec{w} - \vec{c}$$

$$= [3, 0, 4] - \frac{11}{6} [1, -1, 2]$$

$$= \left[\frac{7}{6}, \frac{11}{6}, \frac{2}{6} \right]$$

$$= \frac{1}{6} [7, 11, 2]$$

Ex: Normal form of the plane
 through $A = (1, 2, 3)$, $B = (6, 5, 4)$
 and $C = (3, -3, -3)$?



$$\vec{n} = \vec{AC} \times \vec{AB}$$

$$\vec{AC} = [2, -5, -6]$$

$$\vec{AB} = [5, 3, 1]$$

$$\vec{n} = [13, -32, 31]$$

$$\begin{array}{r} 2 \quad -5 \quad -6 \quad 2 \quad -5 \\ \times \quad \times \quad \times \\ 5 \quad 3 \quad 1 \quad 5 \quad 3 \end{array}$$

Normal form

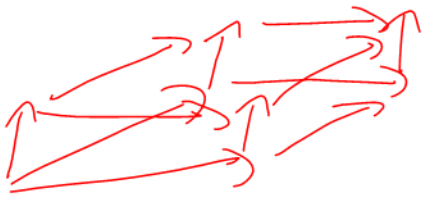
$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} 13 \\ -32 \\ 31 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -32 \\ 31 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

OR

$$\begin{bmatrix} -13 \\ 32 \\ -31 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ 32 \\ -31 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Ex: Volume of the parallelepiped
formed by $\vec{u} = [-4, 2, 3]$, $\vec{v} = [2, 1, 2]$
and $\vec{w} = [3, -3, 6]$?



$$\begin{bmatrix} + & - & + \\ - & + & + \end{bmatrix}$$

$$\text{Volume} = 1 \begin{vmatrix} -4 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & -3 & 6 \end{vmatrix} 1$$

$$= 1 \cdot -4 \begin{vmatrix} 1 & 2 \\ -3 & 6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 3 & 6 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} 1$$

$$= 1 \cdot -4(12) - 2(6) + 3(-9) 1$$

$$= 1 \cdot -87 1$$

$$= 87$$

Ex: Solve using Gauss-Jordan Elimination:

$$\begin{cases} 2x - 4y + 6z = 12 \\ 3x + 5y + z = 7 \\ 10x + 2y + 14z = 38 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 6 & 12 \\ 3 & 5 & 1 & 7 \\ 10 & 2 & 14 & 38 \end{array} \right]$$

$$\frac{R_1}{2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 6 \\ 3 & 5 & 1 & 7 \\ 10 & 2 & 14 & 38 \end{array} \right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 10R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 6 \\ 0 & 11 & -8 & -11 \\ 0 & 22 & -16 & -22 \end{array} \right]$$

$$\frac{R_2}{11} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 6 \\ 0 & 1 & -\frac{8}{11} & -1 \\ 0 & 22 & -16 & -22 \end{array} \right]$$

$$\begin{array}{l} R_1 + 2R_2 \\ R_3 - 22R_2 \end{array} \left[\begin{array}{ccc|c} \overset{x}{\textcircled{1}} & 0 & \frac{17}{11} & 4 \\ 0 & \overset{y}{\textcircled{1}} & -\frac{8}{11} & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} \leftarrow 3 - \frac{16}{11} \\ \uparrow \end{array} \quad \text{RREF}$$

$$z = t$$

$$x + \frac{17}{11}z = 4 \Rightarrow x = 4 - \frac{17}{11}t$$

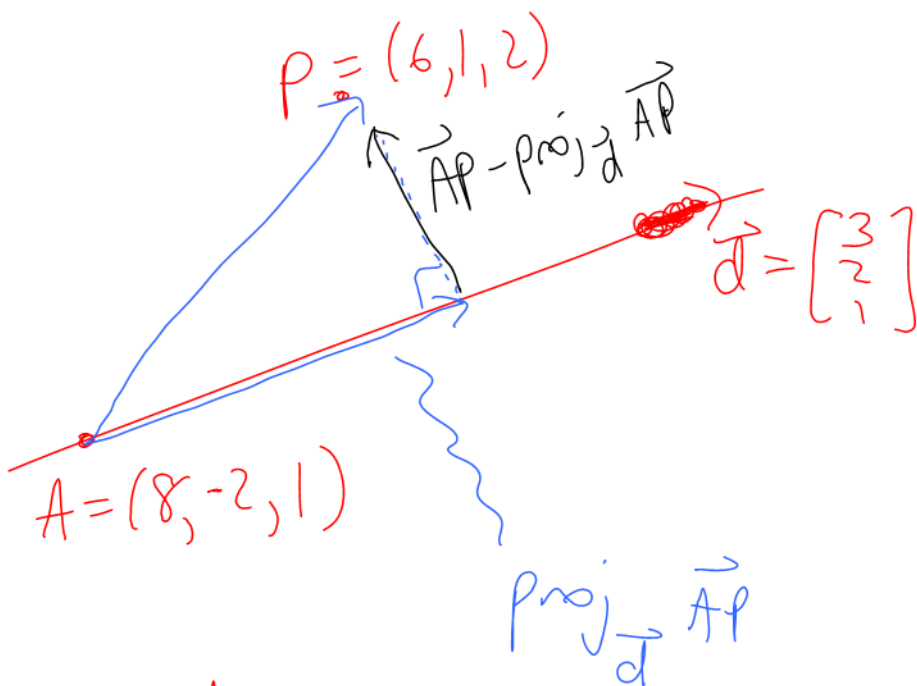
$$y - \frac{8}{11}z = -1 \Rightarrow y = -1 + \frac{8}{11}t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -\frac{17}{11} \\ \frac{8}{11} \\ 1 \end{bmatrix}$$

Ex: Find the distance between

$P = (6, 1, 2)$ and the line

$$\vec{x} = \begin{bmatrix} 8 \\ -2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$



$$\text{distance} = \| \vec{AP} - \text{proj}_{\vec{d}} \vec{AP} \|$$

$$\vec{AP} = [-2, 3, 1]$$

$$\text{proj}_{\vec{d}} \vec{AP} = \frac{\vec{d} \cdot \vec{AP}}{\|\vec{d}\|^2} \vec{d}$$

$$= \frac{1}{14} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{AP} - \text{proj}_{\vec{d}} \vec{AP} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} - \frac{1}{14} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -31/14 \\ 40/14 \\ 13/14 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} -31 \\ 40 \\ 13 \end{bmatrix}$$

$$\text{distance} = \left\| \frac{1}{14} \begin{bmatrix} -31 \\ 40 \\ 13 \end{bmatrix} \right\|$$

$$= \frac{1}{14} \sqrt{2730}$$

Ex: Let $\vec{u} = \begin{bmatrix} x \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

Find x if:

- \vec{u} and \vec{v} are parallel
- \vec{u} and \vec{v} are perpendicular
- the angle between \vec{u} and \vec{v} is 30°

a) $\vec{u} = k\vec{v}$
 $\begin{bmatrix} x \\ 2 \end{bmatrix} = k \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

2nd component: $2 = k(3)$
 $k = \frac{2}{3}$

1st component: $x = k(4)$
 $= \frac{8}{3}$

b) $\vec{u} \cdot \vec{v} = 0$

$$4x + 6 = 0$$

$$x = -\frac{6}{4} = -\frac{3}{2}$$

$$c) \quad \vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$4x+6 = \sqrt{x^2+4} \sqrt{25} \left(\frac{\sqrt{3}}{2}\right)$$

$$8x+12 = 5\sqrt{3} \sqrt{x^2+4}$$

Square Both sides: $64x^2 + 192x + 144$
 $= 75(x^2+4)$

$$64x^2 + 192x + 144 = 75x^2 + 300$$

$$0 = 11x^2 - 192x + 156$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{192 \pm \sqrt{30000}}{22}$$

$$x \approx 16.6, 0.85$$

