Comment: Notice that $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is a solution to the following system:

$$\begin{aligned} x + 2y &= 0 \\ 3x + 4y &= 0 \end{aligned}$$

Definition: A system whose constants are all zero is called a **homogeneous system**. The solution $\vec{x} = \vec{0}$ is called the **trivial solution**.

Fact: A homogeneous system always has at least one solution: $\vec{x} = \vec{0}$.

Example: Consider a homogeneous system with more variables than equations. How many solutions does the system have?

$$m\left\{\underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{n}\right. \left|\begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix}\right] \qquad (n > m)$$

At least 1 solution: $\vec{x} = \vec{0}$

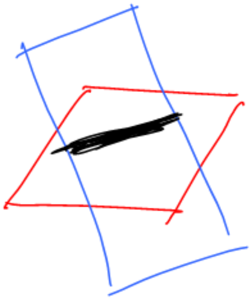
$$\left[\begin{matrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{matrix}\right. \left|\begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix}\right] \quad \text{RREF}$$

At least 1 column without a pivot.

$\Rightarrow$ Infinitely-many solutions.

2.2 #45

Find the intersection of the two planes:



$$\begin{cases} 3x + 2y + z = -1 \\ 2x - y + 4z = 5 \end{cases}$$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & -1 \\ 2 & -1 & 4 & 5 \end{array}\right]$$

$$\frac{R_1}{3} \quad \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ 2 & -1 & 4 & 5 \end{array}\right]$$

$$R_2 - 2R_1 \quad \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{7}{3} & \frac{10}{3} & \frac{17}{3} \end{array}\right]$$

$-1 - \frac{4}{3}$

$$-\frac{3}{7} R_2 \quad \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{10}{7} & -\frac{17}{7} \end{array}\right]$$

$\frac{1}{3} - \frac{2}{3}\left(-\frac{10}{7}\right)$

$$R_1 - \frac{2}{3} R_2 \quad \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & \frac{9}{7} & \frac{9}{7} \\ 0 & 1 & -\frac{10}{7} & -\frac{17}{7} \end{array}\right] \end{array}$$

$-\frac{1}{3} - \frac{2}{3}\left(-\frac{17}{7}\right)$

RREF

$z = t$

$$x + \frac{9}{7} z = \frac{9}{7} \Rightarrow x = \frac{9}{7} - \frac{9}{7} z \Rightarrow x = \frac{9}{7} - \frac{9}{7} t$$

$$y - \frac{10}{7} z = -\frac{17}{7} \Rightarrow y = -\frac{17}{7} + \frac{10}{7} z \Rightarrow y = -\frac{17}{7} + \frac{10}{7} t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{9}{7} \\ \frac{-17}{7} \\ 0 \end{bmatrix} + t \begin{bmatrix} -\frac{9}{7} \\ \frac{10}{7} \\ 1 \end{bmatrix}$$

2.2 #49

Find the point of intersection of $\vec{x} = \vec{p} + s\vec{u}$ and $\vec{x} = \vec{q} + t\vec{v}$, if it exists.

$$\vec{p} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad \vec{q} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \quad \vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{x} = \vec{x}$$

$$\vec{p} + s\vec{u} = \vec{q} + t\vec{v}$$

$$s\vec{u} - t\vec{v} = \vec{q} - \vec{p}$$

$$s\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{array}{cc} s & t \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & -4 \\ 0 & -3 & 0 \\ 1 & -1 & -1 \end{array}\right]$$

$$R_3 - R_1 \quad \left[\begin{array}{cc|c} 1 & -2 & -4 \\ 0 & -3 & 0 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \quad \left[\begin{array}{cc|c} 1 & -2 & -4 \\ 0 & 1 & 3 \\ 0 & -3 & 0 \end{array}\right]$$

$$R_3 + 3R_2 \quad \left[\begin{array}{cc|c} & & \\ 0 & 0 & 9 \end{array}\right]$$

No solution

In $\mathbb{R}^3$ we can have 2 lines that are non-parallel and non-intersecting.

2.2 #29

Solve
$$\begin{array}{cc} r & s \end{array}$$
$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

RREF

no info

$r = 2$

$s = -1$

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$