Test 1

FRI FEB 2

1.1-1.4, 2.1-2.2

5 Questions, with parts

Bring calculator

Bring music/earplugs

Practice Problems on website

Example: Solve by Gauss-Jordan Elimination:

$$\begin{aligned} w + \ x + 2y + 10z &= 5 \\ x + \ y + \ \ z &= 2 \\ w + 3x + 4y + 12z &= 9 \end{aligned}$$

$$\begin{array}{ccccc} w & x & y & z & \# \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 12 & 9 \end{array}\right]$$

$$R_3 - R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 & 4 \end{array}\right]$$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 - 2R_2 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 9 & 3 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \text{RREF}$$

$y = s$

$z = t$

(y and z are free variables

s and t are parameters)

$$w + y + 9z = 3 \implies w = 3 - y - 9z \implies w = 3 - s - 9t$$
$$x + y + z = 2 \implies x = 2 - y - z \implies x = 2 - s - t$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -9 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Example: Find the intersection of the two lines:

$$\vec{x} = \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \text{ and } \vec{x} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{x} = \vec{x}$$

$$\begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} - t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix}$$

$$s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -6 \end{bmatrix}$$

$$\begin{cases} 2s - t = 0 \\ 1s - t = -2 \\ \quad \dots \end{cases}$$

$$\begin{array}{c} \begin{array}{ccc} s & t & \end{array} \\ \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 1 & -1 & -2 \\ -1 & -1 & -6 \end{array}\right] \end{array}$$

$R_1 \Leftrightarrow R_2$
$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 2 & -1 & 0 \\ -1 & -1 & -6 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 + R_1$
$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 1 & 4 \\ 0 & -2 & -8 \end{array}\right]$$

$R_1 + R_2$, $R_3 + 2R_2$
$$\begin{array}{c} \begin{array}{ccc} s & t & \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$s = 2, \; t = 4$

Sub $s = 2$ into 1st line OR $t = 4$ into 2nd line $\qquad \vec{x} = \begin{bmatrix} -1 \\ 8 \\ 3 \end{bmatrix}$

Example: How many solutions does the following system have?

$$\begin{aligned} x+ky &= 1\\ kx+\ y &= 1 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & k & 1\\ k & 1 & 1 \end{array}\right]$$

R_2-kR_1 $$\left[\begin{array}{cc|c} 1 & k & 1\\ 0 & 1-k^2 & 1-k \end{array}\right]$$

$1-k^2\neq 0$

$\dfrac{R_2}{1-k^2}$ $$\left[\begin{array}{cc|c} 1 & k & 1\\ 0 & 1 & \frac{1-k}{1-k^2} \end{array}\right]$$

REF

1 unique solution

$1-k^2=0$

$(1-k)(1+k)=0$

$k=1$ $$\left[\begin{array}{cc|c} 1 & 1 & 1\\ 0 & 0 & 0 \end{array}\right]$$

∞-many solutions

$k=-1$ $$\left[\begin{array}{cc|c} 1 & -1 & 1\\ 0 & 0 & 2 \end{array}\right]$$

no solution

$$\begin{cases} \text{No Solution if } k=-1\\ \infty\text{-many solutions if } k=1\\ \text{1 unique solution if } 1-k^2\neq 0 \end{cases}$$

Definition: The **rank** of a matrix is the number of nonzero rows in its REF or RREF.

solvable

Fact: If a system is consistent then:
rank+(# of parameters in solution)=# of variables

Example: Verify the fact for the following system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

rank = 2
# of parameters in solution = 1
# of variables = 3 ✓

Example: Rephrase the fact in terms of columns of the coefficient matrix.

rank + (# of parameters) = # of variables

(# of columns with pivots) + (# of columns without pivots) = # of columns