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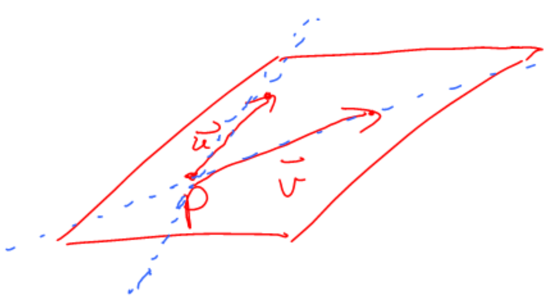
**Part 3. Planes in $\mathbb{R}^3$**

**Example:** Consider the plane through $P = (1, -1, 3)$ with normal $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Describe the plane in both normal and general form.

**Definition:** The **vector form** for a plane in $\mathbb{R}^3$ is $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$ where:
$\vec{u}$ and $\vec{v}$ are nonparallel direction vectors
$s$ and $t$ represent any real numbers




$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$$

**Example:** Consider the plane through $P = (6, 0, 0)$, $Q = (0, 6, 0)$ and $R = (0, 0, 3)$. Describe the plane in vector and parametric form.

Direction vectors $\vec{u} = \overrightarrow{PQ} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$

$\vec{v} = \overrightarrow{PR} = \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$


(non parallel ✓)

Vector $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} + t\begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$$

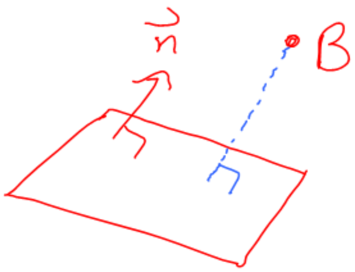
Parametric: $x = 6 - 6s - 6t$, $y = 6s$, $z = 3t$

**Example:** Summarize the twelve descriptions

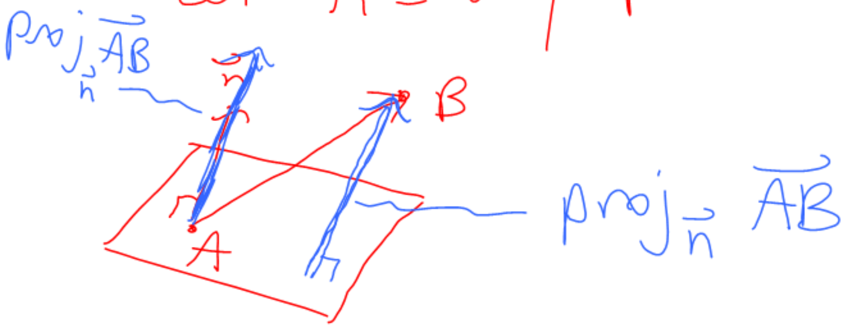
| | Line in $\mathbb{R}^2$ | Line in $\mathbb{R}^3$ | Plane in $\mathbb{R}^3$ |
|---|---|---|---|
| General | $ax + by = c$ | X | $ax + by + cz = d$ |
| Normal | $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$ | X | $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$ |
| Vector | $\vec{x} = \vec{p} + t\vec{d}$ | $\vec{x} = \vec{p} + t\vec{d}$ | $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$ |
| Parametric | $\begin{cases} x = \\ y = \end{cases}$ | $\begin{cases} x = \\ y = \\ z = \end{cases}$ | $\begin{cases} x = \\ y = \\ z = \end{cases}$ |

## Part 4. Geometry Problems

**Example:** Find the distance between $B = (1, 3, 3)$ and the plane $\mathcal{P} : x + y + 2z = 7$




Let A = any point on plane.




$$\text{Distance} = \| \text{proj}_{\vec{n}} \overrightarrow{AB} \|$$

$$A = (7, 0, 0)$$

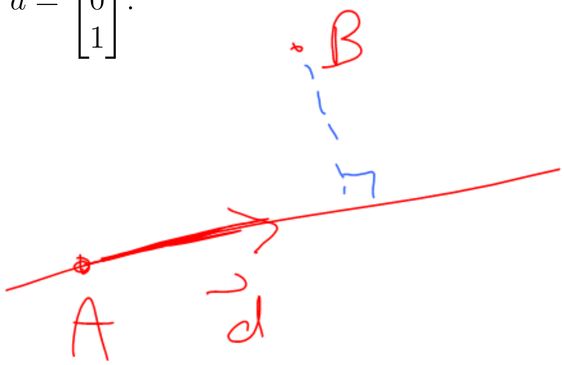
$$\overrightarrow{AB} = [-6, 3, 3]$$

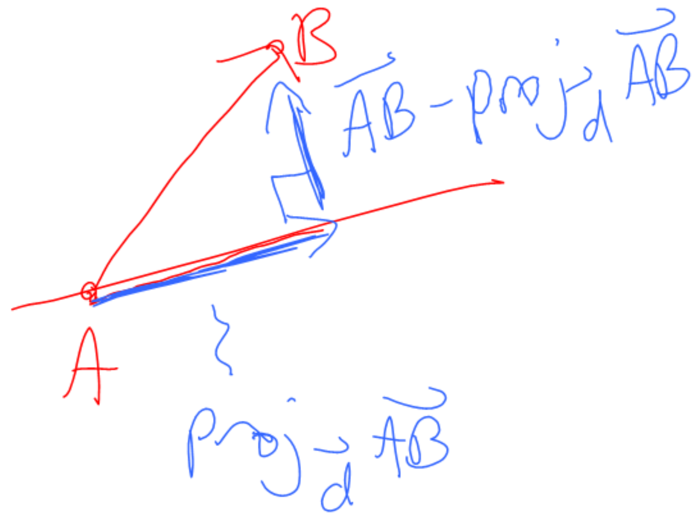
$$\vec{n} = [1, 1, 2]$$

$$\text{proj}_{\vec{n}} \overrightarrow{AB} = \frac{\vec{n} \cdot \overrightarrow{AB}}{\|\vec{n}\|^2} \vec{n} = \frac{3}{6} [1, 1, 2]$$

$$\text{Distance} = \left\| \frac{3}{6} [1, 1, 2] \right\| = \frac{3}{6} \sqrt{6} = \frac{\sqrt{6}}{2}$$

**Example:** Find the distance between $B = (1, 1, 0)$ and the line through $A = (0, 1, 2)$ with $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.




$$\text{Distance} = \left\| \vec{AB} - \text{proj}_{\vec{d}}\,\vec{AB} \right\|$$

$$\vec{AB} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

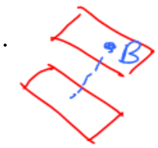
$$\text{proj}_{\vec{d}}\,\vec{AB} = \frac{\vec{d} \cdot \vec{AB}}{\|\vec{d}\|^2}\,\vec{d} = \frac{-1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{AB} - \text{proj}_{\vec{d}}\,\vec{AB} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 \\ 0 \\ -3/2 \end{bmatrix} \text{ or } \frac{3}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

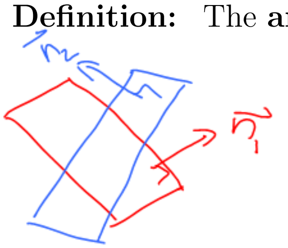
$$\text{Distance} = \left\| \frac{3}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\| = \frac{3}{2}\sqrt{2}$$

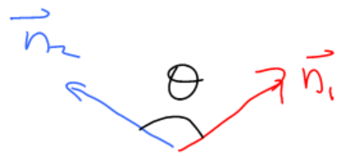
**Comment:** To find the distance between parallel planes, pick a point on one of the planes. Find the distance between that point and the other plane.




**Comment:** To find the distance between parallel lines, pick a point on one of the lines. Find the distance between that point and the other line.


**Definition:** The **angle between planes** is defined as the angle between their normals.




**Definition:** **Parallel planes** have parallel normals.
**Perpendicular planes** have perpendicular normals.

Write an equation that describes
"$\vec{n}_1$ and $\vec{n}_2$ are parallel"

$$\vec{n}_1 = k\,\vec{n}_2 \qquad (k \neq 0)$$

Write an equation that describes
"$\vec{n}_1$ and $\vec{n}_2$ are perpendicular"

$$\vec{n}_1 \cdot \vec{n}_2 = 0$$