21

Cramer's Rule

$$y = \frac{|A_2|}{|A|}$$

$$|A_2| = \begin{vmatrix} 3 & 32 & 4 \\ 5 & 39 & -1 \\ 6 & 38 & 1 \end{vmatrix} \qquad \begin{bmatrix} + & - & + \\ & & \end{bmatrix}$$

$$= 3\begin{vmatrix} 39 & -1 \\ 38 & 1 \end{vmatrix} - 32\begin{vmatrix} 5 & -1 \\ 6 & 1 \end{vmatrix} + 4\begin{vmatrix} 5 & 39 \\ 6 & 38 \end{vmatrix}$$

$$= 3(77) - 32(11) + 4(-44)$$

$$= -297$$

$$|A| = \begin{vmatrix} 3 & -1 & 4 \\ 5 & -2 & -1 \\ 6 & 2 & 1 \end{vmatrix}$$

$$= 3(0) + 1(11) + 4(22)$$

$$= 99$$

$$y = \frac{-297}{99} = -3$$

(24) (Section 4.3)

Recall: If $A\vec{x} = \lambda\vec{x}$ and $\lambda \neq 0$

then $A^{-1}\vec{x} = \lambda^{-1}\vec{x}$.

If $A\vec{x} = \lambda\vec{x}$ then

$$A^n\vec{x} = \lambda^n\vec{x}$$

for non-negative integers n.

Together:

$$A^{-n}\vec{x} = \lambda^{-n}\vec{x} \qquad (\lambda \neq 0)$$

let $c_1\vec{v}_1 + c_2\vec{v}_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$$\begin{array}{cc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 7 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \end{array}\right]$$

$$c_1 = 5, \quad c_2 = -2$$

$$A^{-5}\begin{bmatrix}3\\7\end{bmatrix} = A^{-5}\left(5\begin{bmatrix}1\\1\end{bmatrix} - 2\begin{bmatrix}1\\-1\end{bmatrix}\right)$$

$$= 5A^{-5}\begin{bmatrix}1\\1\end{bmatrix} - 2A^{-5}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$= 5\left(\frac{1}{2}\right)^{-5}\begin{bmatrix}1\\1\end{bmatrix} - 2(-1)^{-5}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$= 160\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}1\\-1\end{bmatrix}$$

$$= \begin{bmatrix}162\\158\end{bmatrix}$$

25 (Section 4.4)

$$P^{-1}AP = D$$

$$AP = PD$$

$$A = PDP^{-1}$$

$$A^n = PDP^{-1}PDP^{-1}\cdots PDP^{-1}$$

$$A^n = P D^n P^{-1}$$

$$A^7 = P D^7 P^{-1}$$

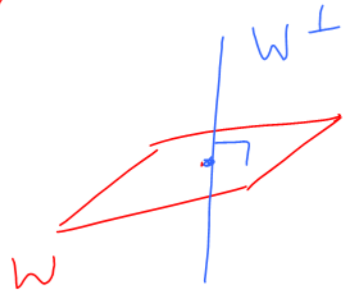
$$= \frac{1}{-8}\begin{bmatrix} -5 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 128 & 0 \\ 0 & -2187 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -3 & -5 \end{bmatrix}$$

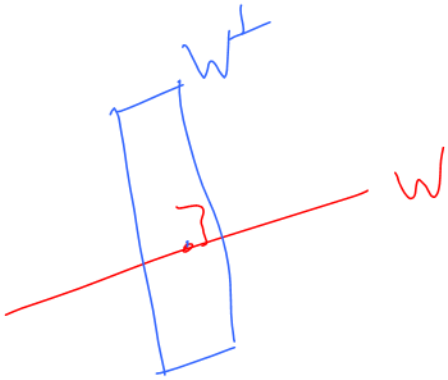
$$= \frac{1}{8}\begin{bmatrix} -5 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 128 & 0 \\ 0 & -2187 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & 5 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -5 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -128 & 128 \\ -6561 & -10935 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -5921 & -11575 \\ -6945 & -10551 \end{bmatrix}$$

29 In $\mathbb{R}^3$





Solve $A\vec{x} = \vec{0}$

A: Basis vectors for W in its <u>rows</u>.

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 2 & 3 & 10 & 7 & 0 \end{array}\right]$$

$$\rightsquigarrow \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -4 & 2 & 0 \\ 0 & 1 & 6 & 1 & 0 \end{array}\right] \end{array}$$

$y = s$

$z = t$

$$\vec{x} = \begin{bmatrix} 4 \\ -6 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} t$$

Basis for $W^{\perp} = \left\{ \begin{bmatrix} 4 \\ -6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$