Suggested HW for Complex Numbers
Omit #7

Omit # 10, 20, 22, 36
from Review Problems

Review

Ex: Find the intersection of $2x+4y+8z=10$ and $2x+5y+5z=7$. Write your answer in parametric form.

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & 4 & 8 & 10 \\ 2 & 5 & 5 & 7 \end{array}\right]$$

$\frac{R_1}{2}$ $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 2 & 5 & 5 & 7 \end{array}\right]$

R_2-2R_1 $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & -3 \end{array}\right]$

R_1-2R_2 $\left[\begin{array}{ccc|c} 1 & 0 & 10 & 11 \\ 0 & 1 & -3 & -3 \end{array}\right]$ RREF

$\uparrow$

$z = t$

$x + 10z = 11 \Rightarrow x = 11 - 10t$

$y - 3z = -3 \Rightarrow y = -3 + 3t$

$$\begin{cases} x = 11 - 10t \\ y = -3 + 3t \\ z = t \end{cases}$$

<u>Ex</u>: Find $\text{proj}_W \vec{u}$ where $W = \text{span}\left(\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\\0\end{bmatrix}\right)$ and $\vec{u} = \begin{bmatrix}1\\2\\4\\1\end{bmatrix}$.

Notice $\left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\\0\end{bmatrix}\right\}$ is <u>not</u> an orthogonal set.

Gram-Schmidt.

Partial Basis $X = \left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}\right\}$

$$\vec{v}_2 = \begin{bmatrix}1\\2\\3\\0\end{bmatrix} - \text{proj}_X \begin{bmatrix}1\\2\\3\\0\end{bmatrix}$$

$$= \begin{bmatrix}1\\2\\3\\0\end{bmatrix} - \frac{4}{3}\begin{bmatrix}1\\0\\1\\1\end{bmatrix}$$

$$3\vec{r}_2 = 3\begin{bmatrix}1\\2\\3\\0\end{bmatrix} - 4\begin{bmatrix}1\\0\\1\\1\end{bmatrix}$$

$$= \begin{bmatrix}-1\\6\\5\\-4\end{bmatrix}$$

Orthogonal Basis for W

$$= \left\{ \underbrace{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}}_{\vec{w}_1}, \underbrace{\begin{bmatrix}-1\\6\\5\\-4\end{bmatrix}}_{\vec{w}_2} \right\}$$

$$\boxed{\vec{u} = \begin{bmatrix}1\\2\\4\\1\end{bmatrix}}$$

$$\text{proj}_W \vec{u} = \text{proj}_{\vec{w}_1} \vec{u} + \text{proj}_{\vec{w}_2} \vec{u}$$

$$= \frac{6}{3}\begin{bmatrix}1\\0\\1\\1\end{bmatrix} + \frac{27}{78}\begin{bmatrix}-1\\6\\5\\-4\end{bmatrix} \checkmark$$

$$\text{or} \quad \frac{1}{26}\begin{bmatrix}43\\54\\97\\16\end{bmatrix} \checkmark$$

<u>Ex</u>: Find the eigenvalues of

$$A = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 + 1 = 0$$

$$1 - 2\lambda + \lambda^2 + 1 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$\lambda = \frac{2 \pm \cancel{\sqrt{-4}}}{2} \quad \cancel{\sqrt{4}\sqrt{-1}} \quad 2i$$

$$\lambda = \frac{2 \pm 2i}{2}$$

$$\lambda = 1 \pm i$$

Ex: Set up and solve a system.

Balance $Al + HCl \rightarrow AlCl_3 + H_2$

$$w(Al) + x(HCl) \rightarrow y(AlCl_3) + zH_2$$

$$\begin{array}{lll}
Al: & w = y & \Rightarrow \quad w - y = 0 \\
H: & x = 2z & \Rightarrow \quad x - 2z = 0 \\
Cl: & x = 3y & \Rightarrow \quad x - 3y = 0
\end{array}$$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right]$$

$\frac{R_3}{-3}$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right]$$

$R_1 + R_3$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right]$$

RREF

$$z = t$$

$$w - \frac{2}{3}z = 0 \implies w = \frac{2}{3}t$$

$$x = 2t$$

...

$$y = \frac{2}{3}t$$

...

Need a non-negative integer solution.

Choose $t = 3$:

$$(w, x, y, z) = (2, 6, 2, 3)$$ ✓

$$2Al + 6HCl \rightarrow 2AlCl_3 + 3H_2$$ ✓

Review Problems

1. $$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$-3 = \sqrt{2}\sqrt{29}\cos\theta$$

$$\frac{-3}{\sqrt{2}\sqrt{29}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-3}{(\sqrt{2}\sqrt{29})}\right)$$

$\approx 113°$

2 $LS = \|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2$

$= (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$

$+ (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$

$= \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$+ \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$= 2(\vec{u}\cdot\vec{u}) + 2(\vec{v}\cdot\vec{v})$

$= 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$

$= RS$

3, 4 Try on your own.

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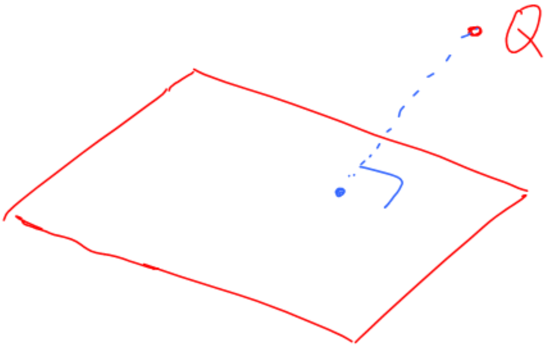
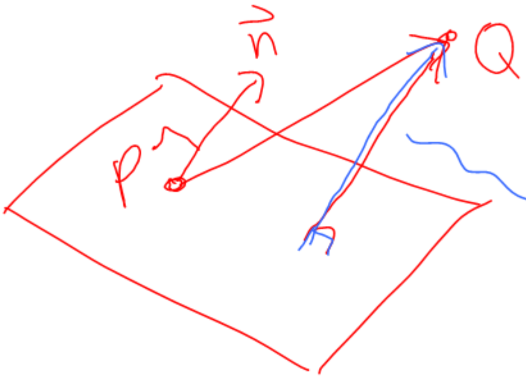




Point P is <u>on the plane.</u>

$\text{proj}_{\vec{n}} \overrightarrow{PQ}$

Distance $= \|\text{proj}_{\vec{n}} \overrightarrow{PQ}\|$

$P = (0,0,0)$

$\overrightarrow{PQ} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$

$\vec{n} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

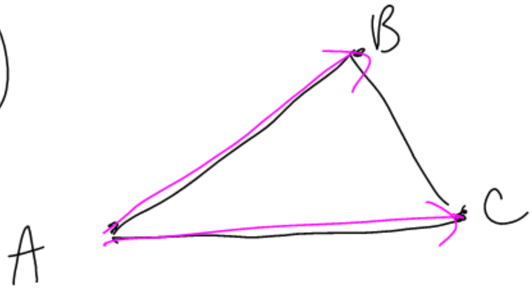
$$\text{proj}_{\vec{n}} \overrightarrow{PQ} = \frac{13}{35} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\text{Distance} = \left\| \frac{13}{35} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\|$$

$$= \frac{13}{35} \left\| \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\|$$

$$= \frac{13}{35} \sqrt{35}$$

(6)



$\vec{AB} = [\;\;,\;\;,\;\;]$

$\vec{AC} = [\;\;,\;\;,\;\;]$

$$\text{area (triangle)} = \frac{\|\vec{AB} \times \vec{AC}\|}{2}$$

Try on your own.