Do Suggested HW for Section 1.1

List of HW Problems and Full Solutions
www.leahhoward.com

Problems on D2L

Vectors are sometimes written in bold font.

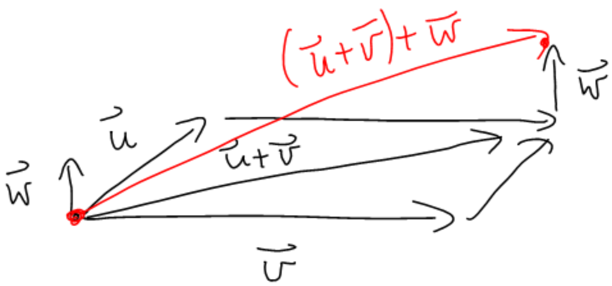
$\mathbf{b}$ is a vector

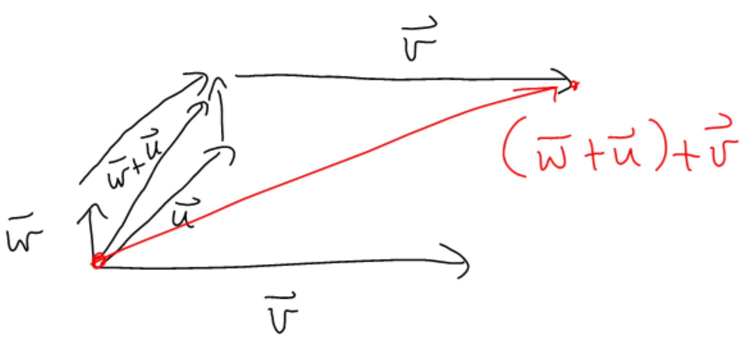
$\vec{b}$ is a vector

b is a real number

Fact: Order doesn't matter when adding vectors. For any vectors $\vec{u}, \vec{v}$ and $\vec{w}$:
$\vec{u} + \vec{v} = \vec{v} + \vec{u}$
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

Example: Let $\vec{u}, \vec{v}$ and $\vec{w}$ be positioned tail to tail to tail. Show geometrically that
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

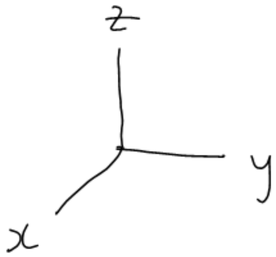


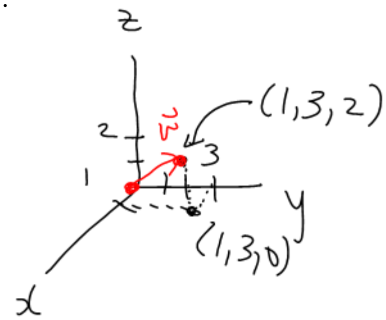


Fact: The above example illustrates that we can write $\vec{u} + \vec{v} + \vec{w}$ without any bracketing.

Definition: Consider the expression: $\vec{v}$ in $\mathbb{R}^n$. This means that $\vec{v}$ has n components, and each component is a real number.

Example: Draw $\vec{v} = [1, 3, 2]$ in $\mathbb{R}^3$.





Definition: The **zero vector** is written $\vec{0}$. Each of its components is zero. The zero vector is useful for algebra.

Example: Write the zero vector in $\mathbb{R}^2$ and $\mathbb{R}^3$.

In $\mathbb{R}^2$: $\vec{0} = [0, 0]$

In $\mathbb{R}^3$: $\vec{0} = [0, 0, 0]$

Example: Let $\vec{u}$ be in $\mathbb{R}^2$. Show (prove) that $\vec{u} + (-\vec{u}) = \vec{0}$.

Let $\vec{u} = [u_1, u_2]$

$$\begin{aligned} \vec{u} + (-\vec{u}) &= [u_1, u_2] + [-u_1, -u_2] \\ &= [0, 0] \\ &= \vec{0} \end{aligned}$$

Example: Solve for $\vec{x}$ given that $7\vec{x} - \vec{a} = 3(\vec{a} + 4\vec{x})$.

$$7\vec{x} - \vec{a} = 3\vec{a} + 12\vec{x}$$
$$-5\vec{x} - \vec{a} = 3\vec{a}$$
$$-5\vec{x} = 4\vec{a}$$
$$\vec{x} = -\frac{4}{5}\vec{a}$$

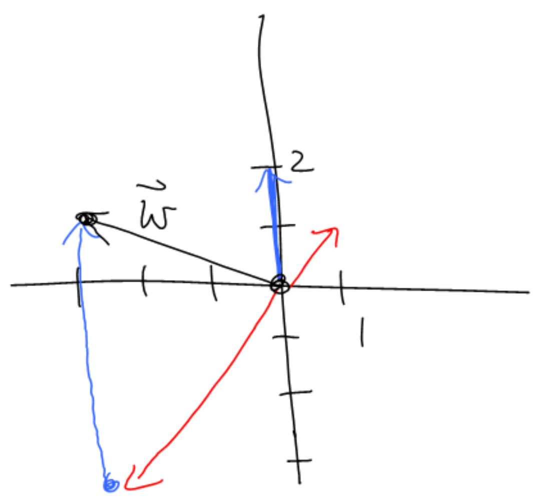
Definition: Consider the statement:
$\vec{w}$ is a **linear combination** of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\2\end{bmatrix}$ with coefficients -3 and 2.
This means that $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$.

Example: Let $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$.

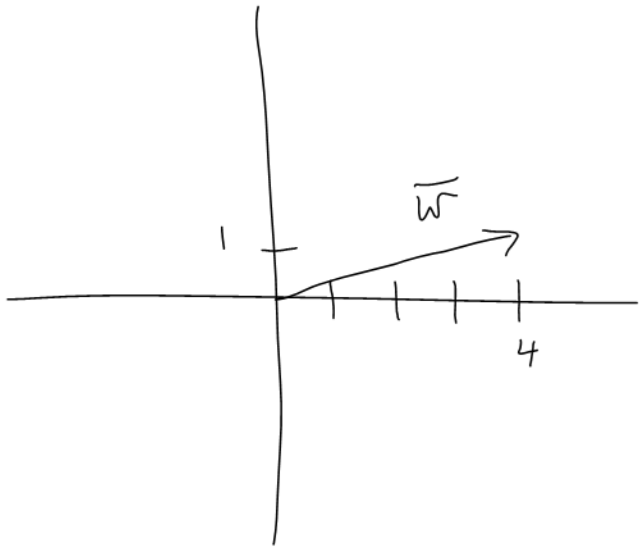
a) Find $\vec{w}$ algebraically.

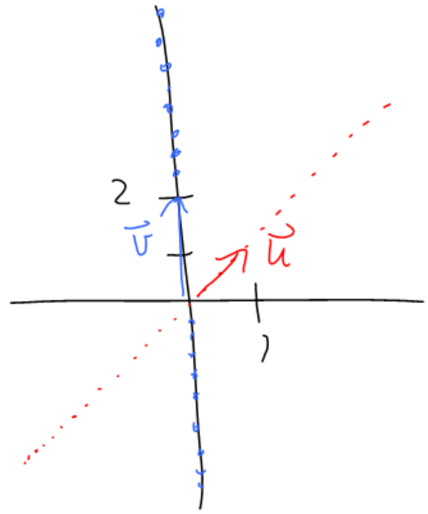
$$\vec{w} = \begin{bmatrix}-3\\-3\end{bmatrix} + \begin{bmatrix}0\\4\end{bmatrix}$$
$$= \begin{bmatrix}-3\\1\end{bmatrix}$$

b) Find $\vec{w}$ geometrically.

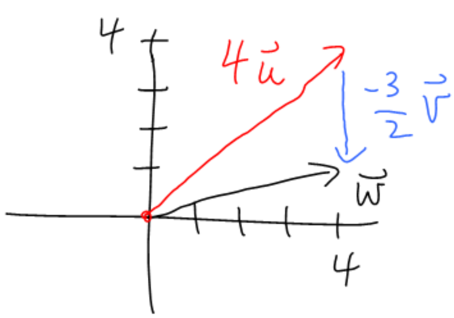


Example: Write $\vec{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ by graphing.





Think of $\vec{u}$ and $\vec{v}$ as the axes.



$$\begin{bmatrix} 0 \\ -3 \end{bmatrix} = k \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$
$$-3 = k(2)$$
$$k = \frac{-3}{2}$$

$$\vec{w} = 4\vec{u} - \frac{3}{2}\vec{v}$$

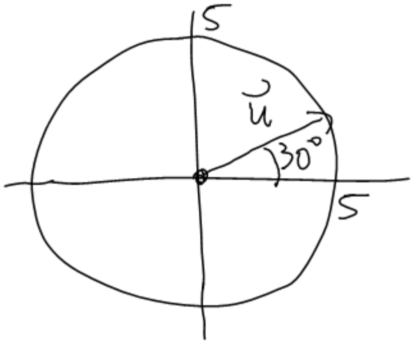
$$\vec{w} = 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

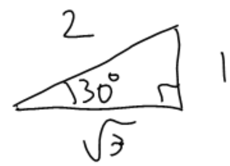
We'll do this algebraically in Ch 2.

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

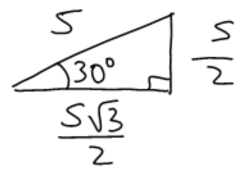
system of equations ...

Example: a) Let $\vec{u}$ be a vector of length 5, in standard position, rotated 30° from the positive x-axis. Find $\vec{u}$ algebraically.



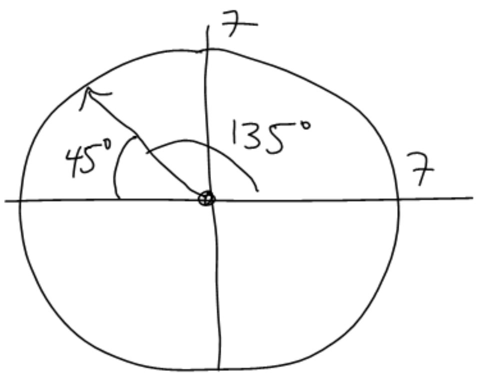


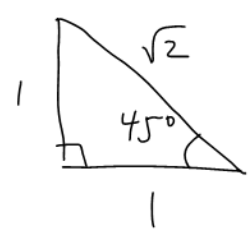
Multiply by $\frac{5}{2}$:



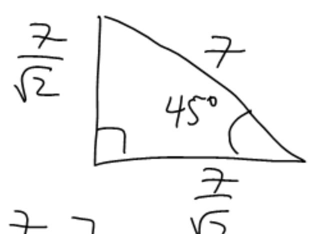
$$\vec{u} = \left[\frac{5\sqrt{3}}{2}, \frac{5}{2}\right]$$

b) Let $\vec{v}$ be a vector of length 7, in standard position, rotated 135° from the positive x-axis. Find $\vec{v}$ algebraically.





Multiply by $\frac{7}{\sqrt{2}}$:



Watch signs! $\vec{v} = \left[-\frac{7}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right]$

Comment: Vectors are often used to represent velocity, acceleration or forces. The vector's direction represents the direction of the velocity/acceleration/force. The vector's length represents the magnitude of the velocity/acceleration/force.

1.2 Length and Angle

Example: Let $\vec{u} = [1, 4, 2, -9]$ and $\vec{v} = [2, 3, -2, -1]$. Calculate the dot product $\vec{u} \cdot \vec{v}$

$$\vec{u} \cdot \vec{v} = 1(2) + 4(3) + 2(-2) + (-9)(-1)$$
$$= 19$$

Example: Calculate:
a) $[1, 5] \cdot [2, -3]$

$= 1(2) + 5(-3)$
$= -13$

b) $[1, 5] \cdot [2, -3, 0]$

undefined

c) $[u_1, u_2] \cdot [u_1, u_2]$

$= u_1(u_1) + u_2(u_2)$
$= u_1^2 + u_2^2$

Fact: Three Properties of the Dot Product
Let $\vec{u}, \vec{v}$ be in $\mathbb{R}^n$. Then:
1) $\vec{u} \cdot \vec{u} \geq 0$
2) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
3) $\vec{u} \cdot \vec{u} = 0$ if and only if $\vec{u} = \vec{0}$

Example: Break Property 3 into two statements, and decide which is more obvious.