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→ Math 251

Skeleton Notes on website
Can print at Satellite Printshop
(CBA Atrium)

Miss a test ⇒ Weight shifts to exam

List of Sugg. HW Problems and Solutions
are on website.
Problems are on D2L.

Course Overview

Matrix Algebra is also known as "Linear Algebra" or "Algebra and Geometry."

A geometry problem could involve visualizing lines and planes in 3D space.

An algebra problem could involve calculating distances and angles, especially in higher dimensions.

Many problems in Matrix Algebra involve the interplay of geometry and algebra.

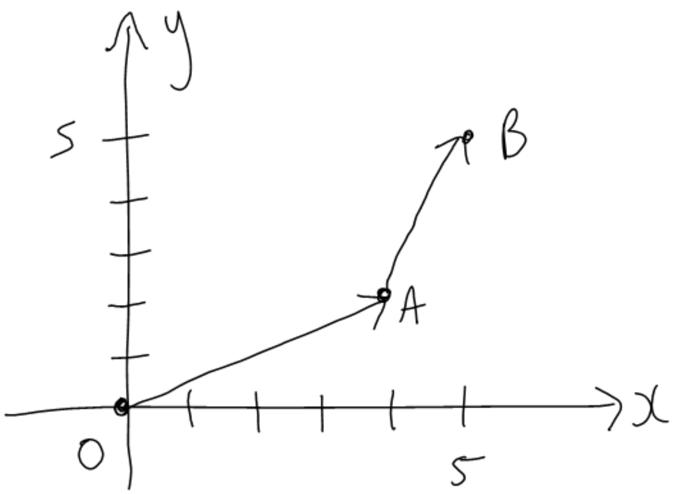
Why do we need higher dimensions? Tracking an object's spatial location and temperature is a 4D problem.

Chapter 1: Vectors

1.1 The Geometry and Algebra of Vectors

Definition: A **vector** is a line segment with direction. Used for velocity, forces etc.

Example: Given $O = (0, 0), A = (4, 2)$ and $B = (5, 5)$. Draw the vectors $\overrightarrow{OA}$ and $\overrightarrow{AB}$. Then write them in component notation.



$\overrightarrow{OA} = [4, 2]$

$\overrightarrow{AB} = [1, 3]$ Think $B-A$

"components"

Alternatively:

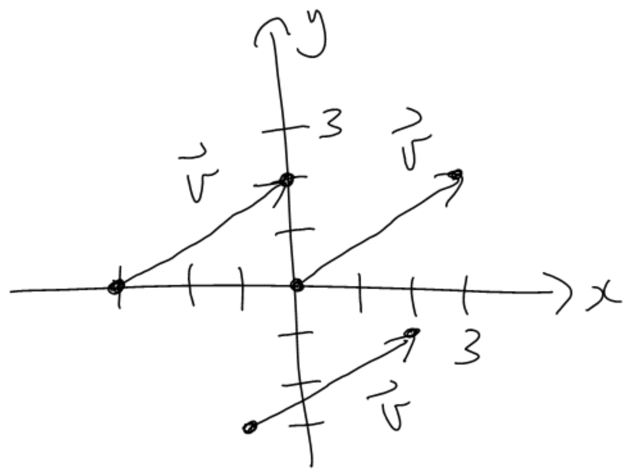
$$\overrightarrow{OA} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\overrightarrow{AB} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Example: Given $C = (-1, -3)$ and $D = (2, -1)$. Find $\vec{v} = \overrightarrow{CD}$ and draw it.

$$\vec{v} = [2-(-1), -1-(-3)]$$ Think $D - C$

$$= [3, 2]$$



Infinitely-many possible answers.

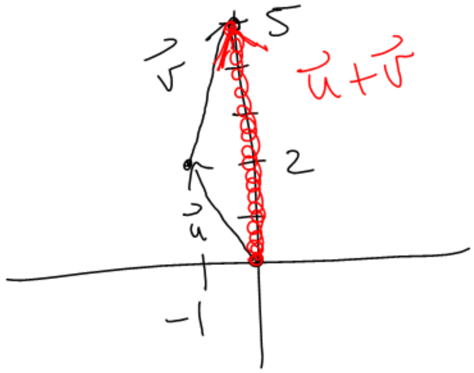
Fact: A given vector can be drawn from any initial position. Rephrased: vectors with the same length and the same direction are considered to be the same vector.

Definition: A vector is in **standard position** if it starts at the origin.

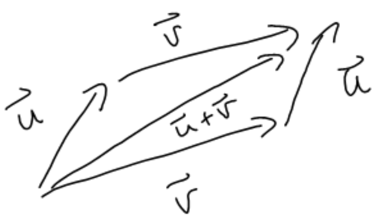
Notation: We use square brackets for vectors and round brackets for points.

Example: Let $\vec{u} = [-1, 2]$ and $\vec{v} = [1, 3]$. Find $\vec{u} + \vec{v}$ both algebraically and geometrically.

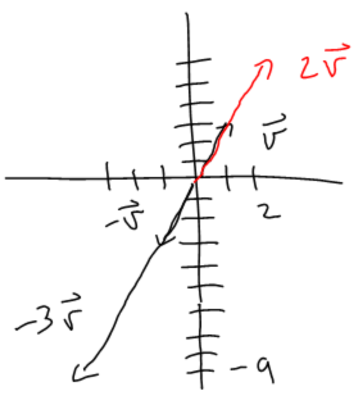
$$\vec{u}+\vec{v} = [-1+1,\ 2+3]$$
$$= [0,\ 5]$$



Example: Graph $\vec{u}, \vec{v}$ and $\vec{u} + \vec{v}$ without a coordinate system.



Example: Let $\vec{v} = [1, 3]$. Graph $2\vec{v}, -\vec{v}$ and $-3\vec{v}$.



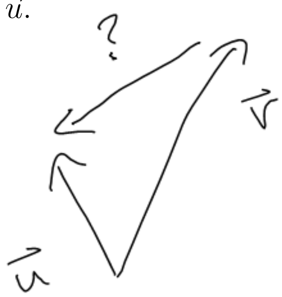
$$2\vec{v} = [2, 6]$$
$$-\vec{v} = [-1, -3]$$
$$-3\vec{v} = [-3, -9]$$

Definition: The process of multiplying a vector by a real number is called **scalar multiplication**. It produces a vector that is parallel to the original vector.

Example: Calculate $[2, 6] - [3, 4]$

$$= [2, 6] + [-3, -4]$$
$$= [2-3, 6-4]$$
$$= [-1, 2]$$

Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Find the vector that runs from the head of $\vec{v}$ to the head of $\vec{u}$.



? = backwards along $\vec{v}$
then forwards along $\vec{u}$

$$= -\vec{v} + \vec{u}$$
$$= \vec{u} - \vec{v}$$

Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Draw the parallelogram formed by $\vec{u}$ and $\vec{v}$. Label the four diagonals.